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Internal Quality Assurance Cell (IQAC)

3.3.1 - Links to the papers published in Journals listed in UGC CARE List

2017– 22

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INDEXING OF JOURNALS WITH PUBLICATIONS

S.No.	Name of the Journal	Details of UGC Approval / Indexing
1	*Communications in Statistics-Theory and Methods*	SCIE, Scopus, Web of Sciences
2	*Journal of Statistical Computation and Simulation*	SCIE, Scopus, Web of Sciences
3	International Journal of Mathematical Archive (IJMA)	Approved Journals,Journal No. (No. Assigned in the Approved List of Journals), Up to 2-May-2018 (Journal No.-48371) (Search by Journal
4	Global Journal of Pure and Applied Mathematics	indexed in SCOPUS(2010-2016), the Mathematical Reviews, MathSciNet, and EBSCO Databases, **ICI, Index Copernicus**
5	Mathematics	SCOPUS database and Web of Science
6	Journal of the Applied Mathematics, Statistics and Informatics (JAMSI)	Web of Science
7	Mathematics and Statistics	Scopus database
8	Journal of Applied Probability and Statistics	Scopus
9	Proceedings of STATPARCONF, Statistical Paradigms Recent Advances and Reconciliations	Scopus
10	European Journal of Pure and Applied Mathematics	Scopus and Web of Science
11	Thai Journal of Mathematics	Scopus and Web of Science
12	Thailand Statistician	Scopus
13	JOURNAL OF NEW RESULTS IN SCIENCE (JNRS)	UGC Approval
14	YMER	Scopus, UGC CARE
15	Chilean Journal of Statistics	Scopus and Web of Science
16	Journal of Emerging Technologies and Innovative Research (JETIR)	http://jetir.org/jetir%20ugc%20approval.pdf
17	International Journal of Recent Scientific Research	http://recentscientific.com/approved-ugc
18	International Journal of Creative Research Thoughts (IJCRT	https://ijcrt.org/ugc%20approval.jpg
19	International Journal of Applied Engineering Research	UGC Approved Journal - 2017 (Journal No. - 64529)
20	Science, Technology and Development	https://ugccare.unipune.ac.in/Apps1/User/Web/CloneJournalsGroupII
21	NOVYI MIR Research Journal	https://ugccare.unipune.ac.in/Apps1/User/Web/CloneJournalsGroupII
22	Dogo Rangsng Research Journal	https://ugccare.unipune.ac.in/Apps1/Content/Files/pdf/DogoRangsangResearchJournalOriginal.pdf
23	International Journal for Research in Engineering Application & Management (IJREAM)	https://www.ijream.org/UGC/Welcome%20to%20UGC,%20New%20Delhi,%20India.html

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3.3.1 Number of research papers published per teacher in the Journals notified on UGC website during the last five years

Papers published in UGC - CARE Listed Journals

S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link to the recognition in UGC enlistment of the		
							Link to website of the Journal	Link to article / paper / abstract of the article	Is it listed in UGC Care list
1	*Health Insurance in Andhra Pradesh: Dr. NTR Vaidya Seva Health Insurance Scheme*	Smt. V. SailajaVani	Department of Commerce	Journal of Emerging Technologies and Innovative Research	2020	ISSN-2349-5162	www.jetir.org	https://www.jetir.org/papers/JETIR1905A72.pdf	http://jetir.org/jetir%20ugc%20approval.pdf
2	*Application of Fuzzy Quasi Metric Spaces in the Domain of Words*	Dr. B. Rami Reddy	Department of Mathematics	Journal of Emerging Technologies and Innovative Research (JETIR)	2019	ISSN-2349-5162	www.jetir.org	https://www.jetir.org/view?paper=JETIR1905D04	http://jetir.org/jetir%20ugc%20approval.pdf
3	*The Sum of i^{th} Smallest Parts of r - partitions of n*	Dr. K. Hanuma Reddy Lt. Dr. A. Majusree	Department of Mathematics	International Journal of Recent Scientific Research	2020	ISSN: 0976-3031	http://www.recentscientific.com	DOI: 10.24327/IJI	http://recentscientific.com/approved-ugc
4	*The number of i^{th} smallest parts of of r - partitions n*	Dr. K. Hanuma Reddy Lt. Dr. A. Majusree	Department of Mathematics	International Journal of Creative Research Thoughts (IJCRT	2020	ISSN: 2320-2882	www.ijcrt.org	https://ijcrt.org/archivelist.php	https://ijcrt.org/ugc%20approval.jpg
5	*On Semicircular Extreme-value distribution*	Dr. S.V.S. Girija Sri R. Srinivas	Department of Mathematics	International Journal of Applied Engineering Research	2019	ISSN 0973-4562	https://www.ripublication.com/ijaer.htm	https://www.ripublication.com/ijaer19/ijaerv14n9_18.pdf	UGC Approved Journal - 2017 (Journal No. - 64529)

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							Link to website of the Journal	Link to article / paper / abstract of the article	Is it listed in UGC Care list
6	*Stability and Validity Indicating UV Spectrophotometric Rapid Assay method for the Estimation of Promethazine Theoclate*	Dr. T. Umamaheswara Rao	Department of Chemistry	Science, Technology and Development	2021	0950-0707	https://journals	https://drive.google.com/file/d/1xJtOCeFSSkrVKx9IoZ2P3wct7XXwZzrw/view	https://ugccare.unipune.ac.in/Apps1/User/Web/CloneJournalsGroupII
7	*Stability and Validity Indicating Assay Method for Estimation of Multidrug components of Etophylline and Theophylline Anhydrous –Forced Degradation Studies*	Dr. T. Umamaheswara Rao	Department of Chemistry	NOVYI MIR Research Journal	2021	0130-7673	https://novyimir.net/	https://novyimir.net/volume-5-issue-4-2020/	https://ugccare.unipune.ac.in/Apps1/User/Web/CloneJournalsGroupII
8	*Outcomes Bases Education : Planning, Teaching and Assessment Possibilities*	Smt. Ch. Aruna	Department of English	Dogo Rangsng Research Journal	2020	2347-7180	https://journal-dogorangsang.in/	https://journal-dogorangsang.in/no_1_Dec_20/14.pdf	https://ugccare.unipune.ac.in/Apps1/Content/Files/pdf/DogoRangsangResearchJournalOriginal.pdf
9	*Unsteady Hall effects on magneto hydrodynamics flow through a permeable Channel*	Dr. Y. Uday Kumar	Department of Mathematics	International Journal for Research in Engineering Application & Management (IJREAM)	Sep 2020	ISSN : 2454-9150	https://www.ijream.org/	Non digital	https://www.ijream.org/UGC/Welcome%20to%20UGC,%20New%20Delhi,%20India.html

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							Link to website of the Journal	Link to article / paper / abstract of the article	Is it listed in UGC Care list
10	*Marshall – Olkin Stereographic Circular Logistic Distribution*	Dr. S.V.S. Girija Sri Y. Sreekanth	Department of Mathematics	YMER	2022	0044-0477	https://ugccare.unipune.ac.in/Apps1/User/Web/CloneJournalsGroupII	https://ymerdigital.com/uploads/YMER210692.pdf	https://ugccare.unipune.ac.in/Apps1/User/Web/CloneJournalsGroupII

HEALTH INSURANCE IN ANDHRA PRADESH: Dr. NTR VAIDYA SEVA HEALTH INSURANCE SCHEME

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***Abstract**: Recently health care and health insurance and health care finances gained much attention in India. Various states in India and central government has been taken attention and putting grater efforts to introduce health care insurance for the poor and below poverty families for their quality medical care and providing financial protection against very high medical expenditures. The successfully running Dr.NTR Vaidya Seva Health Insurance scheme in Andhra Pradesh among the different schemes implementing in the various states and government of Andhra Pradesh earlier similar scheme. The Insurance scheme covered around 18 lakh families are benefited during 2014 to 2018 in 13 districts of the state. The scheme started with 938 procedures covered earlier and has been gradually extended to 1044 procedures. The majority of beneficiaries utilizing the scheme are illiterate and have a rural address. Since inception of the scheme (02.06.2014) to December 2018 – various Medical camps were held by the network hospitals in rural areas and total Surgeries/Therapies done by under this scheme is 1764855worth of more than 4,632 crore in which government is 437691 and corporate hospitals 1327164.*

***Key words: NTR Vaidya Seva,** Health Insurance, Community Health insurance, Healthcare, Healthcare Expenditure.*

I. Introduction

Health is an important for all it is not personnel and also it has impact on society. Health is real wealth for everybody it is not only increase the quality of human life but also decreases both public and private expenditure on ill health or sickness and diseases. Health is a very significant constituent of Human resource development, and it also declared as a fundamental human right. Health care services can provide various factors to reduce infant mortality rate, death rate, raise life expectancy with keeping diseases under control. Economic growth of any country depends upon for ways by World development report 1993 they are: Reduction of production caused by illness of workers, it permits the natural recourses of that has been totally or practically increasable because of their diseases, it also effect the enrolment of children in schools, makes them better able to learn. It is frees for another use of resources that would otherwise has to be spent on treating illness. The economic gains are relatively better for poor people, who are in general most handicapped by ill health. Who stand to gain the most from the improvement of utilization of their personal resources.

I.

The out-of pocket expenditure/payments (OOPs) is defined as the direct expenditure which is paid by individuals for their health care when they are in services. The families spend less than 10% or more than 10% for their total family income which is measure the indication of catastrophic health expenditure [1]. The increase in catastrophic health expenditure it causes those families fall in poverty [2]. The total health expenditure for India was figured as Rs.4,83,259 crores for the years 2014-15 by the report of National Health account (NHA).

Total Health Expenditure (THE) includes of current and capital expenditures incurred by Government and private sources with External/Donor funds. Around 62.6% households are affected by OOPE was Rs. 3,02,425 cores during the year of 2014-15 In India [2]. Catastrophic health expenditure is more in rural India (25.3%) when compared with urban (17.5%) [3]. Out of pocket expenditure was 87% of the rural poverty occurs in poor states and this was 67% of richest states in India [4]. The poverty percentage is very high 3.5% in rural areas when compared to urban areas 2.5% [5,6]. A large proportion of the population sells their assets for their inpatient care [7,9]. Catastrophic out-of-pocket expenditure was reduced by the individual households with the help of their health insurance provide financial protection in the event of their ill health. Low income and developing countries health care finance is still mainly based on out-of pocket (OOP) payments, and the lack of prepayment mechanisms like insurance. In the absence of insurance, an illness not only reduces welfare directly and it also increases risk of economical failure due to their high treatment expenditures [9,10]. The historical timelines of health Insurance in India were given Table 1. It is evident that from 1907 to 2018 different schemes were implemented for better health for the people of India.

Table 1. Historical timeline of Health Insurance in India

Year	Name of the health insurance scheme
1907	First general insurance company
1952	Employees State Insurance Scheme Implemented (ESI Act 1948)
1954	Central Government Health Scheme
1973	General Insurance Corporation: 4 public insurers – National, New India, Oriental and United India
1999	Establishment of Insurance Regulatory Development Authority 100% Foreign Direct Investment In Health Insurance
2003	Yeshasvini Health Insurance, Karnataka
2007	De-tariffication of insurance
2007	Rajiv Arogyasri Scheme (RAS), Andhra Pradesh
2008	GOI ’s Rashtriya Swasthya Bima Yojana (RSB Y)
2009	Kalaignar, Tamil Nadu
2010	RSB Y Plus, Himachal Pradesh and Vajpayee Arogyasri Scheme (VAS), Karnataka
2014	Arogyasri Scheme, Telangana
2014	Dr.NTR Vaidya Seva
2016	Dr.NTR Arogya Rasha
2018	Ayushman Bharat - India’s National Health Protection Mission

Objectives of the study

1. To explore the current status of major health insurance schemes in India
2. To study the role of A.P state health insurance scheme (NTR Vaidya Seva) in Andhra Pradesh.

II. Methodology

The data collected from various sources such as scheme website, available assessment reports, and the data provided by the Dr. NTR Vaidya Seva Trust and research reports.

III Background

Current status of health financing including health insurance in India

Public health expenditure in India (total of central and state governments) remained constant at approximately 1.3% of gross domestic product (GDP) between 2008 and 2015 and increased marginally to 1.4% in 2016-17. Total public health expenditure in India (both central and state governments) is approximately 1.3% of GDP (gross domestic product) between2008-2015 and it’s increased slightly 1.4% in

2016-2017. For the years 2018-19 it is estimated at 3.9% in which both public and private sectors. The total health expenditure allocation is estimated by Ministry of family welfare about 54.600 crore. **Table 2** shows the various states health related programmes and their coverage amount.

Table 2. Publicly financed health insurance schemes in India

State	Programme name	Primary care	Secondary care	Tertiary care	Maximum benefit (Rs.)
Maharashtra	Mahatma Jyotiba Phule Jan Arogya Yojana (MJPJAY)	No	No	Yes	1,50,000/
Gujarat	Mukhyamantri Amrutam Yojana (MAJ)	No	Partly	Yes	2,00,000/
Chhattisgarh	Sanjeevani Kosh	No	No	Yes	3,00,000/
Chhattisgarh	Chief Minister Child Heart Protection Scheme	No	No	Yes	1,80,000
Kerala	RSBY-CHIS (Comprehensive Health Insurance Scheme)	No	Yes	Yes	30,000/- (RSBY)+ 70,000/-
Karnataka	Yeshasvini Co-operative Farmers Healthcare Scheme	No	Yes	Yes	2,00,000/-
Andhra Pradesh	RAS (Rajiv Arogya Sri) Community Health Insurance Scheme (CHIS) upto 2014	No	Yes	Yes	2,00,000/
	Dr.NTR Vaidya Seva From 2014 onwards	No	Yes	Yes	2,50,000/
Telangana	Arogya Sri Community Health Insurance Scheme (CHIS)	No	Yes	Yes	2,00,000/-
Tamil Nadu	CM Health Insurance Scheme	No	Yes	Yes	1,00,000/-
Himachal Pradesh	RSBY plus	No	No	Yes	1,75,000
Meghalaya	Megha Health Insurance Scheme (MHIS)	Partial	Yes	Yes	1,60,000/-
Assam	Atal Amrit Abhiyan	No	Yes	Yes	2,00,000/-

The National Health Mission received the highest allocation at Rs.30,130 crore and constituted 55% of the total allocation. According to the National Family Health Survey 4 (2015- 16) (Ministry of Health and Family Welfare 2017), only 29% of households in India have one member covered under any health insurance scheme, be it public or private (20% women and 23% men). The top five states according to coverage are Andhra Pradesh (75%), Chhattisgarh (69%), Telangana (66%), Tamil Nadu (64%) and Tripura (58%). In-patient hospitalization expenditure in India has increased nearly 300% during the last ten years (National Sample Survey Office 2015). Household health expenditures include out-of-pocket expenditures (OOPE) (95%) and insurance (5%). According to the National Health Accounts (2014-15), total OOPE is 3.02 lakh crore. The highest OOPE is made towards purchasing medicines— 1.30 lakh crores. (43%), followed by private hospitals—86,189 crores (28%).

Government Sponsored Health Insurance Schemes and their Key Features

Revenue collection, risk pooling and purchasing of insurance scheme are three key functions for scheme performance and its functioning. The source of funds and collection mechanisms are important to pool up the revenue collection. While pooling funds refer to build up and running of funds, to ensure that financial risk to pay health care is borne by all and not by individuals who fall ill. Third function is purchasing care which refers to paying for health care. The insurer and their health insurance or the organizer of the scheme purchases services on behalf of a population. All these schemes broadly involve contracting with providers of designing an appropriate benefit package and making the choices around paying for them [11].

Dr. Nandamuri Taraka Rama Rao Vaidya Seva Health Insurance Scheme (Dr NTR Vaidya Seva Health Insurance Scheme) of Andhra Pradesh

According to Dr.Nandamuri Taraka Rama Rao Vaidya Seva Health Insurance Scheme is introduced Government of Andhra Pradesh to attainment of universal health coverage for BPL (Below Poverty Line) families. This scheme is helps to achieve equity, providing accountable and evidence-based good-quality health-care services in the state to assist poor families from catastrophic health expenditure. The scheme is a unique PPP model in the field of health insurance; tailor made to the health needs of poor patients and provides end-to-end cashless services for identified diseases under secondary and tertiary care through a network of service providers from Government and private sector.

The scheme is designed in such a way that the benefit in the primary care is addressed through free screening and outpatient consultation both in the health camps and in the network hospitals as part of scheme implementation. The IEC activity during the health camps, screening, counseling and treatment of common ailments in the health camps and out-patient services in network hospitals are supplementing the government health care system in preventive and primary care. The scheme is designed to get benefit for the people to identify for their primary care addressed through the free screening and outpatient consultation both in the health camps and network hospitals as part of scheme implementation. The main activity during the health camps, screening, counseling and treatment for common ailments are conducted by both government and network hospitals for preventive and primary care. The organization of the scheme and effective implementation, the state government established Dr.NTR vaidya seva trust. This trust works under the chairmanship of the Honorable Chief Minister and administrative chief executive officer who is an IAS or IPS officer. The other members are specialists in health care management's personals. The organization chart is given in Figure 1.

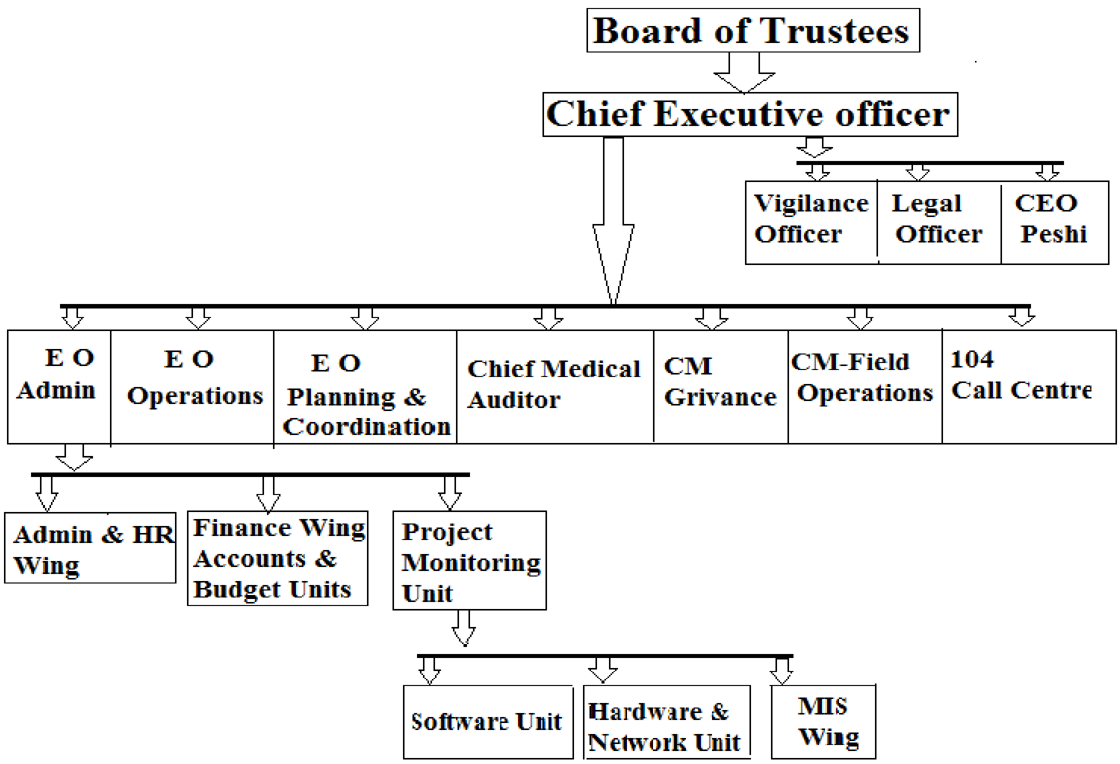

Figure 1.The organization chart of NTR Vaidya Seva

The main objectives are (i) To provide free quality hospital care for families who effected, by purchase of quality medical services from identified network of health care providers through a self-funded reimbursement mechanism (serviced by Trust). (ii) To provide financial safety against the catastrophic health expenditures (iii) To strengthen the government hospitals through require areas of financing. (iv) To provide universal coverage of health for both urban and rural poor in the State of Andhra Pradesh.

Geometry of health insurance coverage

- Population coverage (Breadth of Universal Health coverage): The beneficiaries of this scheme are the members of Below Poverty Line (BPL) families. They are enumerated and photographed in White Ration Card linked with Adhar card and available in Civil Supplies Department database.
- Financial coverage (Height of Universal Health coverage): The scheme shall provide coverage for the services to the beneficiaries up to Rs.2.50 lakh per family per annum on floater basis. There shall be no co-payment under this scheme.
- Benefit Coverage (Depth of Universal Health coverage).

Out-Patient: The scheme is designed to get benefit in the primary care is addressed through free screening and outpatient consultation both in the health camps and network hospitals as part of scheme implementation.
In-patient: The scheme shall provide coverage for the 1044 "Listed Therapies" for identified diseases in the 29 categories – which are listed in www.ntrvaidyaseva.ap.gov.in

Package includes the following services:

- End-to-end cashless service offered through a network hospitals from the time of reporting of a patient till ten days post discharge medication, including complications if any up to thirty (30) days post-discharge, for those patients who undergo a "listed therapy(ies).
- Free outpatient evaluation of patients for listed therapies who may not undergo treatment for "listed therapies.
- All the pre-existing cases under listed therapies are covered under the scheme.
- Providing Food and Transportation for them.

Follow-up Services

Follow-up services are provided for a period of one year through fixed packages to all the patients whoever required for long term follow-up therapy. In order to get optimum benefit for the patients by the follow up service the procedure and avoid complications. Follow-up package for consultation, investigations, drugs etc., for one year for listed therapies are formulated by Technical committee of the Trust in consultation with specialists. More details we can find in www.ntrvaidyaseva.ap.gov.in.

Performance of the Scheme:

The impact of Dr.NTR vaidya seva scheme was evaluated by a rapid assessment, commissioned by the government of Andhra Pradesh. The aim of the present assessment was to explore the contribution of the scheme to the reduction of catastrophic health expenditure among the poor and to suggest ways by which relief of the scheme could be enhanced. This novel scheme was beginning to reach the BPL households in 13 districts of the Andhra Pradesh providing access to free secondary and tertiary healthcare to seriously ill poor people. An integrated model encompassing primary, secondary and tertiary care would be of larger number of families benefited below the poverty line and more cost-effective for the government. There is considerable potential for the government to build on this successful start and to strengthen equity of access and the quality of care provided by the scheme.

From Table 3 it is evident from the period 2nd June 2014 to March 2018, the total number of therapies preauthorized under NTR Vaidya Seva scheme is 17,64,855 and the expenditure was paid for 405.66, 814.49, 994.85, 1326.42 and 1089.79 cores for 2014 to 2018 respectively. Total amount paid for preauthorized therapies for all the years under this scheme is Rs.4631.21 Cores and average amount per therapy is Rs.26,241. The preauthorize number is increasing every year from 2014 to 2017. There is a slight decrease in 2018 year due to another scheme government introduced Arogrya Rasha for the people. Total preauthorized therapies were done in Government and corporate hospitals are given in table 3 it is evident from the data both government and corporate hospitals preauthorized therapies are increased from 2014 to 2018 it is shown in Figure 2. The average expenditure was shared nearly 25 % government and 75 % corporate hospitals.

Table 3 Year wise preauthorized therapies and expenditure sharing

Year	Total number of Preauthorized	Preauthorized Amount paid from NTR Vaidya Seva Trust. Rs. Cores	Average amount Rs.	Govt	Corp	Sharing	
						Govt. %	Corp. %
2014	1,60,948	405.66	25205	43176	117772	26.83	73.17
2015	3,21,671	814.49	25320	83907	237764	26.08	73.92
2016	3,63,204	994.85	27391	82503	280701	22.72	77.28
2017	4,79,688	1326.42	27652	110342	369346	23.00	77.00
2018	4,39,344	1089.79	24805	117763	321581	26.80	73.20
Total	17,64,855	4631.21	26241	437691	1327164	24.80	75.20

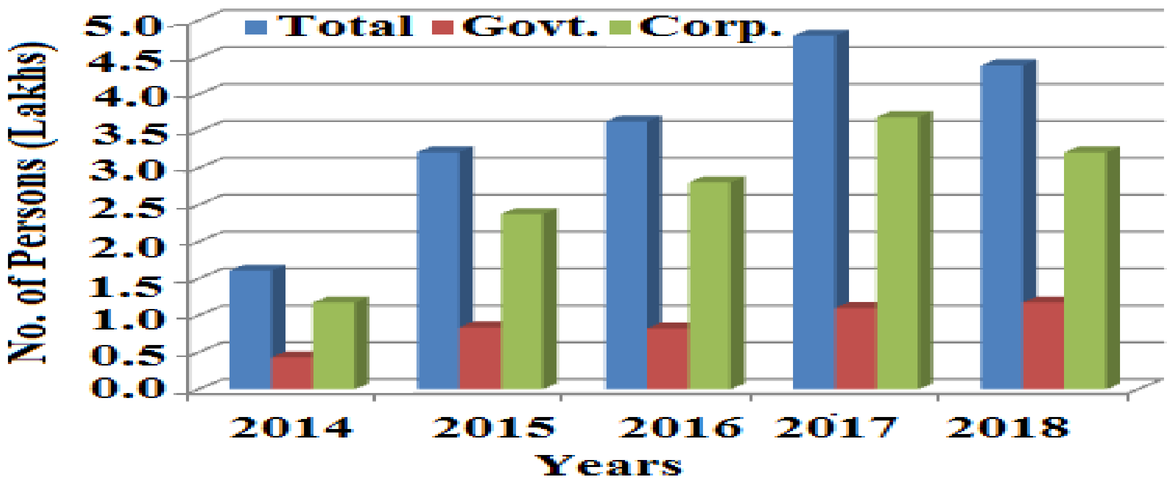

Figure 2 year wise performance of government and corporate hospitals

Table 4 District – wise total number of therapies preauthorized under NTR Vaidya Seva.

DISTRICT	2014	2015	2016	2017	2018	Total	Average per year	%
Anatapur	8522	18732	21309	28854	28997	106414	21282.8	6.03
Kurnool	10819	21196	24771	32013	33765	122564	24512.8	6.94
YSR Kadapa	9407	18765	20829	29244	29539	107784	21556.8	6.11
Chittoor	12238	23218	25478	36790	35503	133227	26645.4	7.55
Nellore	13404	26827	27034	39836	33233	140334	28066.8	7.95
Prakasam	11304	22844	26312	34819	30363	125642	25128.4	7.12
Guntur	17674	33265	38769	50416	46589	186713	37342.6	10.58
Krishna	16451	29857	32450	41964	35394	156116	31223.2	8.85
West Godavari	14450	27909	31898	40995	34022	149274	29854.8	8.46
East Godavari	18207	37094	45992	55398	52585	209276	41855.2	11.86
Vishakhapatnam	11842	25093	27561	36481	34528	135505	27101	7.68

Srikakulam	8587	18750	21018	27079	22326	97760	19552	5.54
Vizianagaram	8043	18121	19783	25799	22500	94246	18849.2	5.34
Total	160948	321671	363204	479688	439344	1764855	352971	

Table 4 shows 13 Districts and each district wise total number of preauthorized under NTR vaidya seva scheme from 2014 to 2018. It has been found that the total number of preauthorized that has been taken under this scheme is 17,64,855. The average of the each year about 3,52,971 therapies for all the districts are carried out. Srikakulam district average is low, where as highest average is East Godavari district 41855.2. Highest number of therapies preauthorized in East Godavari district 11.86 percent (209276), Guntur district 10.58 percent (186713) and Krishna district 8.85 percent (156116). Lowest number of therapies preauthorized in Vizinagaram district 5.34 percent (94246) and Srikakulam district 5.54 percent (97760).

In the Andhra Pradesh total 13 districts the gender and male, female, male child and female child are gradually increased from 2014 to 2018. Figure 3 shows the total therapies are highest for 2017 for male (260727) and female (178968). The gender therapies were given in Table 5.

Table 6 explains the category of the people for their therapies and average performance of the NTR Vaidya seva scheme. Maximum numbers of beneficiaries are BC category next followed by OC and SC category, very less in number by other category.

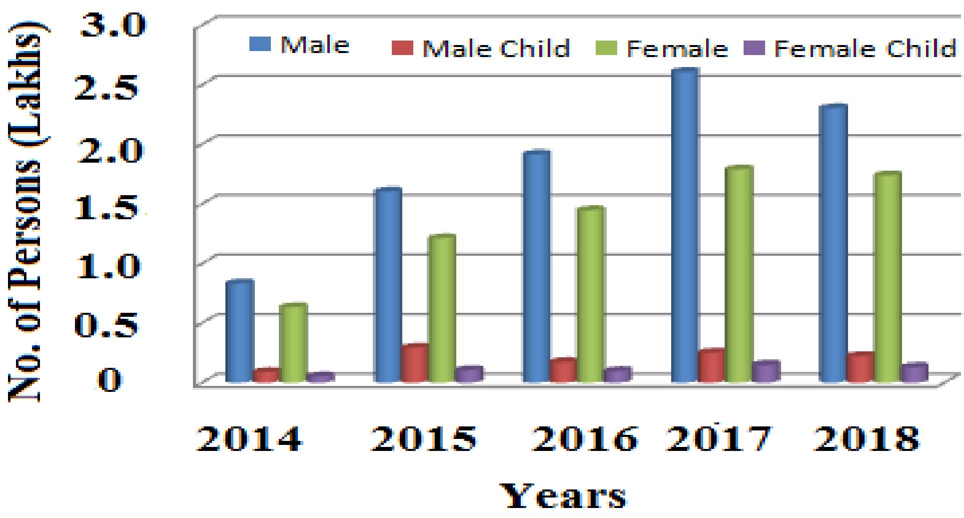

Figure 3 Gender therapies for the years 2014 to 2018.

Table 5 .The total gender therapies of the NTR Vaidya seva scheme.

Year	Male	Male Child	Female	Female Child
2014	83151	9027	63428	5342
2015	160504	29243	121215	10709
2016	191613	17162	144562	9867
2017	260727	25068	178968	14925
2018	230247	22202	173853	13042
Total	926242	102702	682026	53885

Table 6 Various Categories of NTR Vaidya Seva for the years 2014 to 2018.

Year	OC	BC	SC	ST	Minorities	Others
2014	40089	26981	79270	4348	10132	128
2015	79358	159733	52747	9265	20302	266

2016	88701	183604	58869	10112	21572	346
2017	114123	246164	76742	13138	29150	371
2018	99008	229994	70921	12592	26580	249
Total	**421279**	**846476**	**338549**	**49455**	**107736**	**1360**
Percentage	**23.87**	**47.96**	**19.18**	**2.80**	**6.10**	**0.08**

Conclusion

NTR Vaidya Seva Scheme is extremely popular in Andhra Pradesh State for the poor and BPL families. More than 17.64 lakh families are benefited during 2014 to 2018. The corporate hospitals are highest share when compare to government hospitals. The follow up service is also quite good in rating. But there is very less provision for outpatient treatment of everyday illnesses like BP, Blood sugar. Which affect the working expenditure for the patient and effect the families increases out of pocket expenses still continue. So, there is a coming up need to enlarge the scope of the scheme and better health care by plugging loopholes, in the interest of society and social well being of people, especially the poor and downtrodden. Without proper health care, the economic and social development of any country becomes impossible. In view of above key comments here and Government of Andhra Pradesh is planning to distribute free medicines for the poor people for everyday illnesses through health workers. However NTR Vaidya seva scheme is undoubtedly regarded as a boon to poor and downtrodden families in entire 13 districts. Who are otherwise vulnerable to a variety of diseases and efficient treatment due to heavy health expenditure.

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APPLICATION OF FUZZY QUASI METRIC SPACES IN THE DOMAIN OF WORDS

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ABSTRACT
In this paper we prove Banach fixed point theorem in intuitionistic fuzzy quasi-metric space. The existence of a solution for a recurrence equation which appears in the average case analysis of Quicksort algorithms is obtained as an application. We generalize the results of Romaguera, Sapena and Tirado and also generalize several known results.

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Keywords : fuzzy quasi-metric, , contraction mapping, fixed point, recursive equation.

1.1 Introduction

Fuzzy set theory was first introduced by Zadeh in 1965 to describe the situations where data are uncertain. Thenafter the concept of fuzzy sets was generalized as intuitionistic fuzzy set by Atanassov [1,2] in 1984, has a wide range of applications in various fields.
With the help of continuous t-norm the concept of fuzzy metric space was modified by Kramosil and Michalek [3] and George and Veeramani [4]. The concept of fuzzy quasi-metric space was introduced by Gregori and Romaguera [5] by generalizing the concept of fuzzy metric space given by Kramosil and Michalek [6].
The concept of intuitionistic fuzzy quasi-metric space was introduced by Tirado [7] by generalizing the notion of intuitionistic fuzzy metric space given by Alaca, Turkoglu and Yildiz [8] to the quasi-metric setting and gave intuitionistic fuzzy quasi-metric version of the Banach contraction principle. Our basic references are [9], [10], [11], [12], [13], [7].
In [11], Grabiec proved fuzzy versions of celebrated Banach fixed point theorem and Edelstein fixed point theorem. Romaguera, Sapena and Tirado [16] proved the Banach fixed point theorem in fuzzy quasi-metric spaces and applied the result to the domain of words.

1.2 Preliminaries

Definition 1.2.1. A binary operation $\star : [0,1] \times [0,1] \to [0,1]$ is a continuous $t - norm$ if it satisfies the following conditions:

(1) $\star$ is commutative and associative.
(2) $\star$ is continuous.
(3) $a \star 1 = a$ for all $a \in [0,1]$:
(4)$a \star b \leq c \star d$ whenever $a \leq c$ and $b \leq d$ with $a,b,c,d \in [0,1]$

Definition 1.2.2. A binary operation $\nabla : [0,1] \times [0,1] \to [0,1]$ is a continuous $t - conorm$ if it satisfies the following conditions:

(1) ∇ is commutative and associative.
(2) ∇ is continuous.
(3) $a \nabla 0 = a$ for all $a \in [0,1]$:
(4) $a \nabla b \leq c \nabla d$ whenever $a \leq c$ and $b \leq d$ with $a,b,c,d \in [0,1]$

Remark 1.2.1. The concept of triangular norms $(t - norm)$ and triangular conforms $(t - conorm)$ are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions respectively. These concepts were originally introduced by Menger [17] in his study of statistical metric spaces. Several examples for these concepts were purposed by many authors.

Definition 1.2.3. A 5-tuple $(X, M, N, \star, \nabla)$ is said to be an intuitionistic fuzzy quasi-metric space if X is an arbitrary set, $\star$ is a continuous t-norm, ∇ is a continuous t-conorm and M and N are fuzzy sets on $X^2 \times [0,1)$ satisfying the following conditions:

(1) $M(x,y,t) + N(x,y,t) \leq 1,\ \forall\, x,y \in X$ and $t > 0$:
(2) $M(x,y,0) = 0\ \forall\ x,y \in X$:
(3) $M(x,y,t) = M(y,x,t) = 1,\ iff\ x = y\ \forall\, t > 0$:
(4)$M(x,y,t) \star M(y,z,s) \leq M(x,z,t+s)\ \forall\, x,y,z \in X$ and $s,t > 0$:
(5) for all $x,y \in X$, $M(x,y,.)\colon [0,1) \to [0,1]$ is left continuous.
(6) $N(x,y,0) = 1\ \forall\, x,y \in X$:
(7) $N(x,y,t) = N(y,x,t) = 0\ iff\ x = y\ \forall\, t > 0$:
(8) $N(x,y,t)\ \nabla\ N(y,z,s) \geq N(x,z,t+s)\ \forall\, x,y,z \in X$ and $s,t > 0$:
(9) for all $x,y \in X, N(x,y,.)\colon [0,1) \to [0,1]$ is right continuous.

In this case, we say that $(M, N, \star, \nabla)$ is an intuitionistic fuzzy quasi-metric (an ifqm) on X. If in addition M and N satisfy $M(x,y,t) = M(y,x,t)$ and $N(x,y,t) = N(y,x,t)$ for all $x,y \in X$ and $t > 0$ then $(M, N, \star, \nabla)$ is called intuitionistic fuzzy metric on X and $(X, M, N, \star, \nabla)$ is called an intuitionistic fuzzy metric space.

Example 1.2.1. Let (X,d) be a quasi-metric space. Define $t-norm\ a \star b = min\{a,b\}$ and $t-conorm\ a\ \nabla\ b = max\{a,b\}$ and for all $x,y \in X$ and $t > 0$, $M_d(x,y,t) = \frac{t}{t+d(x,y)}$ *and* $N_d(x,y,t) = \frac{d(x,y)}{t+d(x,y)}$.

Then $(X, M, N, \star, \nabla)$ is an intuitionistic fuzzy quasi-metric space. We call this intuitionistic fuzzy quasi-metric (M,N) induced by the metric d the standard intuitionistic fuzzy-quasi metric. Furthermore it is easy to check that $(M_d)^{-1} = M_{d^{-1}}$, $(M_d)^i = M_{d^i}$, $(N_d)^{-1} = N_{d^{-1}}$, $(N_d)^i = N_{d^i}$. The topology τ generated by d coincides with the topology τ_{MN_d} generated by the induced intuitionistic fuzzy quasi-metric $(M, N, \star, \nabla)$.

Remark 1.2.2. It is clear that if $(X, M, N, \star, \nabla)$ is an ifqm-space then $(X, M, \star)$ is a fuzzy quasi-metric space. Conversely if $(X, M, \star)$ is a fuzzy quasi-metric space on X, then $(X, M, 1-M, \star, \nabla)$ is an ifqm-space where $a\ \nabla\ b = 1 - [(1-a) \star (1-b)]$ for all $a,b \in [0,1]$. If $(M, N, \star, \nabla)$ is an ifqm on X, then $(M^{-1}, N^{-1}, \star, \nabla)$ is also an ifqm on X where M^{-1} and N^{-1} are the fuzzy sets in $X \times X \times (0,1)$ defined by

$$M^{-1}(x,y,t) = M(y,x,t) \text{ and } N^{-1}(x,y,t) = N(y,x,t).$$

Moreover if we denote M^i and N^s, the fuzzy sets on $X^2 \times [0,1)$ given by

$$M^i(x,y,t) = min\{M(x,y,t), M^{-1}(x,y,t)\}$$

and

$$N^s(x,y,t) = max\{N(x,y,t), N^{-1}(x,y,t)\}.$$

Then $(M^i, N^s, \star, \nabla)$ is an intuitionistic fuzzy metric on X. In order to construct a suitable topology on an ifqm-space $(X, M, N, \star, \nabla)$ it seems natural to consider balls $B(x,r,t)$ defined similarly to Park [16] and Alaca, Turkoglu and Yildiz [1] by

$B(x,r,t) = \{y \in X : M(x,y,t) > 1 - r. N(x,y,t) < r\ \forall\ x \in X\}\ r \in (0,1)$ and $t > 0$:

Then one can prove as in Park [16] that the family of sets of the form

$\{B(x,r,t)\colon x \in X, 0 < r < 1, t > 0\}$ is a base for the topology $\tau_{M,N}$ on X.

Definition 1.2.4. Let $(X, M, N, \star, \nabla)$ be an intuitionistic fuzzy metric space. A sequence $\{x_n\}_n\ in\ X$ is called a Cauchy if for each $\epsilon \in (0,1)$ and each $t >$, there exists $n_0 \in N$ such that $M(x_n, x_m, t) > 1 - \epsilon$, $N(x_n, x_m, t) < \epsilon$

whenever $n, m \geq n_0$. We say that $(X, M, N, \star, \nabla)$ is complete if every Cauchy sequence is convergent.

Definition 1.2.5. A sequence $\{x_n\}_n$ in an intuitionistic fuzzy metric space $(X, M, N, \star, \nabla)$ is said to be converges to a point $x \in X$ if and only if $\lim_{n\to\infty} M(x, x_n, t) = 1$ and $\lim_{n\to\infty} N(x, x_n, t) = 0$ for all $t > 0$.

Definition 1.2.6. Let $(X, M, N, \star, \nabla)$ be an intuitionistic fuzzy metric space. A sequence $\{x_n\}_n$ in X is called G −Cauchy if for each $p \in N$ and each $t > 0$,
$\lim_{n\to\infty} M(x_n, x_{n+p}, t) = 1$ and $\lim_{n\to\infty} N(x_n, x_{n+p}, t) = 0$.
We say that $(X, M, N, \star, \nabla)$ is G −complete if every G −Cauchy sequence is convergent.

1.3. The Banach Fixed Point Theorem In Intuitionistic Fuzzy Quasi-Metric Space

Definition 1.3.1. A B −contraction on an intuitionistic fuzzy metric space $(X, M, N, \star, \nabla)$ is a self mapping f on X such that there is a constant $k \in (0,1)$ satisfying

$$M(f(x), f(y), kt) \geq M(x, y, t)$$

and $N(f(x), f(y), kt) \leq N(x, y, t)$ for all $x, y \in X,\ t > 0$.

Theorem A. Let $(X, M, N, \star, \nabla)$ be a G −complete intuitionistic fuzzy metric space and $f : X \to X$ be a self-map such that

$$M(f(x), f(y), kt) \geq M(x, y, t)$$

and $N(f(x), f(y), kt) \leq N(x, y, t)$,

for all $x, y \in X,\ t > 0$ with $k \in (0,1)$. Then f has a unique fixed point.

Generalizing in a natural way the notations of G-completeness and B-contraction to intuitionistic fuzzy quasi-metric spaces we introduce the following concepts.

Definition 1.3.2. A sequence $\{x_n\}_n$ in an intuitionistic fuzzy quasi-metric space $(X, M, N, \star, \nabla)$ is said to be G −Cauchy if it is a G −Cauchy sequence in the intuitionistic fuzzy metric space $(X, M^i, N^s, \star, \nabla)$.

Definition 1.3.3. An intuitionistic fuzzy quasi-metric space $(X, M, N, \star, \nabla)$ is called G −bicomplete if the intuitionistic fuzzy metric space $(X, M^i, N^s, \star, \nabla)$ is G-complete. In this case we say that $(M, N, \star, \nabla)$ is a fuzzy quasi-metric on X.

Definition 1.3.4. A B −contraction on an intuitionistic fuzzy quasi-metric space $(X, M, N, \star, \nabla)$ is a self mapping f on X such that there is a constant $k \in (0,1)$ satisfying $M(f(x), f(y), kt) \geq M(x, y, t)$**.** Let $(X, M, N, \star, \nabla)$ be a G −bicomplete intuitionistic fuzzy quasi-metric space such that $\lim_{n\to\infty} M(x, y, t) = 1$ and $\lim_{n\to\infty} N(x, y, t) = 0$
for all $x, y \in X$. Then every B −contraction on X has a unique fixed point and $N(f(x), f(y), kt) \leq N(x, y, t)$, for all $x, y \in X, t > 0$.
The number k is called a contraction constant of f.

Theorem 1.3.1

Proof. Let $f : X \to X$ be a B −contraction on X with contraction constant $k \in (0,1)$. Then

$$M(f(x), f(y), kt) \geq M(x, y, t)$$

and

$$N(f(x), f(y), kt) \leq N(x, y, t),$$

for all $x, y \in X, t > 0$.

It immediately follows that

$M^i(f(x), f(y), kt) \geq M^i(x, y, t)$ and
$N^s(f(x), f(y), kt) \leq N^s(x, y, t)$,

for all $x, y \in X, t > 0$: Hence f is a B −contraction on the G −complete fuzzy metric space $(X, M^i, N^s, \star, \nabla)$ and by $Theorem\ A$, f has a unique fixed point.

Definition 1.3.4. A generalized B −contraction on an intuitionistic fuzzy quasi-metric space $(X, M, N,\star,\nabla)$ is a self mapping f on X such that there is a constant $k \in (0,1)$ satisfying

$$M(f(x), f(y), kt) \geq min\{M(x,y,t), M(x, f(x), t), M(y, f(y), t)\}$$
$$and \quad N(f(x), f(y), kt) \leq max\{N(x,y,t), N(x, f(x), t), N(y, f(y), t)\}$$

for all $x, y \in X,\ t > 0$.

The number k is called a contraction constant of f .

Proof. Let $f : X \to X$ be a weak B −contraction on X with contraction constant $k \in (0,1)$. Then

$$M(f(x), f(y), kt) \geq min\{M(x,y,t), M(x, f(x), t), M(y, f(y), t)\}$$
$$and \quad N(f(x), f(y), kt) \leq max\{N(x,y,t), N(x, f(x), t), N(y, f(y), t)\}$$

for all $x, y \in X, t > 0$.

It immediately follows that

$$M^i(f(x), f(y), kt) \geq min\{M^i(x,y,t), M^i(x, f(x), t), M^i(y, f(y), t)\}$$
$$and$$
$$N^s(f(x), f(y), kt) \leq max\{N^s(x,y,t), N^s(x, f(x), t), N^s(y, f(y), t)\}$$

for all $x, y \in X, t > 0$: Hence f is a weak B −contraction on the G −complete fuzzy metric space $(X, M^i, N^s,\star,\nabla)$ and by *Theorem A*, f has a unique fixed point.

1.4. G-Bicompleteness In Non-Archimedean Intuitionistic Fuzzy Quasi-Metric Space

Definition 1.4. 1. An intuitionistic fuzzy quasi-metric space $(X, M, N,\star,\nabla)$ is called a non-Archimedean intuitionistic fuzzy quasi-metric space if $(M, N,\star,\nabla)$ is a non-Archimedean intuitionistic fuzzy quasi-metric on X, that is, $M(x,y,t) \geq min\{M(x,z,t), M(z,y,t)\}$

$and \ N(x,y,t) \leq max\{M(x,z,t), M(z,y,t)\}$,

for all $x, y, z \in X$ and $t > 0$.

Lemma 1.4.1. Each G −Cauchy sequence in a non-Archimedean intuitionistic fuzzy quasi-metric space is a Cauchy sequence.

Proof. Let$\{x_n\}$ be a G −Cauchy sequence in the non-Archimedean intuitionistic fuzzy quasi-metric space $(X, M, N,\star,\nabla)$, then for each $t > 0$, we have $\lim_{n\to\infty} M(x_n, x_{n+1}, t) = 1$ and $\lim_{n\to\infty} N(x_n, x_{n+1}, t) = 0$, which implies that, for each $\epsilon \in (0,1)$, there is $n_0 \in N$ such that

$M^i(x_n, x_{n+1}, t) > 1 - \epsilon$ and $N^s(x_n, x_{n+1}, t) < \epsilon$, for each $n \geq n_0$,

Now let $m > n \geq n_0$. Then $m = n + j$, for some $j \in N$. So

$$M^i(x_n, x_m, t) \geq min\left\{\begin{matrix} M^i(x_n, x_{n+1}, t), M^i(x_{n+1}, x_{n+2}, t), \\ \dots, M^i(x_{n+j-1}, x_{n+j}, t) \end{matrix}\right\}$$
$$> 1 - \epsilon$$

and

$$N^s(x_n, x_m, t) < max\left\{\begin{matrix} N^s(x_n, x_{n+1}, t), N^s(x_{n+1}, x_{n+2}, t), \\ \dots, N^s(x_{n+j-1}, x_{n+j}, t) \end{matrix}\right\}$$
$$< \epsilon.$$

We conclude that $\{x_n\}$ is a Cauchy sequence in $(X, M, N,\star,\nabla)$.

Theorem 1.4.1. Each bicomplete non-Archimedean intuitionistic fuzzy quasi-metric space is G −bicomplete.

Proof. Let $\{x_n\}$ be a G −Cauchy sequence in the bicomplete non-Archimedean intuitionistic fuzzy quasi-metric space $(X, M, N,\star,\nabla)$: By *Lemma* 4.1, $\{x_n\}$ is a Cauchy sequence in $(X, M, N,\star,\nabla)$. Hence there is $x \in X$ such that

$\lim_{n\to\infty} M(x, x_n, t) = 1$ and $\lim_{n\to\infty} N(x, x_n, t) = 0$, for all $t > 0$.

We conclude that

$(X, M^i, N^s,\star,\nabla)$ is G −complete, that is, $(X, M, N,\star,\nabla)$ is G −bicomplete.

Corollary 1.4.2. Each complete non-Archimedean intuitionistic fuzzy metric space is G-complete.

1.5. Application To The Domain Of Words

Let Σ be a non-empty alphabet. Let Σ^{∞} be the set of all finite and infinite sequences ("words") over Σ, where we adopt the convention that the empty sequence ϕ is an element of Σ^{∞}. The symbol $\sqsubseteq$ denote the prefix order on Σ^{∞} , that is, $x \sqsubseteq y \Leftrightarrow x$ is a prefix of y. Now, for each $x \in \Sigma^{\infty}$ denote by $l(x)$ the length of x. Then $l(x) \in [1,\infty)$ whenever $x \neq \phi$ and $l(\phi) = 0$. For each $x, y \in \Sigma^{\infty}$ let $x \sqcap y$ be the common prefix of x and y. Thus the function $d_{\sqsubseteq}$ defined on $\Sigma^{\infty} \times \Sigma^{\infty}$ by

$$d_{\sqsubseteq}(x,y) = \begin{cases} 0 & if \;\; x \sqsubseteq y \\ 2^{-l(x \sqcap y)} & otherwise \end{cases}$$

is a quasi-metric on Σ^{∞} (We adopt the convention that $2^{\infty} = 0$). Actually, $d_{\sqsubseteq}$ is a non-Archimedean quasi-metric on Σ^{∞} and the non-Archimedean quasi metric $(\Sigma^{\infty})^{s}$ is the Baire metric on Σ^{∞}, that is, $(\Sigma^{\infty})^{s}(x,x) = 0$ and $(\Sigma^{\infty})^{s}(x,y) = 2^{-l(x \sqcap y)}$ for all $x, y \in \Sigma^{\infty}$ such that $x \neq y$. It is well known that $(\Sigma^{\infty})^{s}$ is complete. From this fact it is clear that Σ^{∞} is bicomplete. The quasi-metric Σ^{∞} , which was introduced by Smyth [20], will be called the Baire quasi-metric. Observe that condition $d_{\sqsubseteq}(x,y) = 0$ can be used to distinguish between the case that x is a prefix of y and the remaining cases.

Example 1.5.1. Let $d_{\sqsubseteq}$ be a (non-Archimedean) quasi-metric on a set X and let $M_{d_{\sqsubseteq}}$ *and* $N_{d_{\sqsubseteq}}$ are fuzzy sets in $X \times X \times [0,1)$ given by

$$M_{d_{\sqsubseteq}}(x,y,t) = \frac{t}{t + d_{\sqsubseteq}(x,y)} \quad \text{and}$$

$$N_{d_{\sqsubseteq}}(x,y,t) = \frac{d_{\sqsubseteq}(x,y)}{t + d_{\sqsubseteq}(x,y)}$$

for all $x, y \in X$ and $t > 0$. Then $(M_{d_{\sqsubseteq}}, N_{d_{\sqsubseteq}}, \wedge, \vee)$ is a (non-Archimedean) intuitionistic fuzzy quasi-metric on X, where $\wedge$ denotes the continuous

t −norm and $\vee$ denotes the continuous t −conorm given by $a \wedge b = min\{a,b\}$ and $a \vee b = max\{a,b\}$.

It is clear that $\tau_{MN_{d_{\sqsubseteq}}} = \tau_{d_{\sqsubseteq}}$ and that $(M_{d_{\sqsubseteq}}, N_{d_{\sqsubseteq}}, \wedge, \vee)$ is bicomplete if and only if $(X, d_{\sqsubseteq})$ is bicomplete.

Proposition 1.5.1. $(\Sigma^{\infty}, M_{d_{\sqsubseteq}}, N_{d_{\sqsubseteq}}, \wedge, \vee)$ is a G −bicomplete non-Archimedean intuitionistic fuzzy quasi-metric space.

Proposition 1.5.2. $(\Sigma^{\infty}, M_{d_{\sqsubseteq}}, N_{d_{\sqsubseteq}}, \wedge, \vee)$ is a G −bicomplete non-Archimedean intuitionistic fuzzy quasi-metric space.

The intuitionistic fuzzy non-Archimedean quasi-metric $(M_{d_{\sqsubseteq}}, N_{d_{\sqsubseteq}}, \wedge, \vee)$ is given by $M_{d_{\sqsubseteq 1}}(x,y,0) = 0$ and $N_{d_{\sqsubseteq 0}}(x,y,0) = 1$ for all $x, y \in \Sigma^{\infty}$.

$M_{d_{\sqsubseteq 1}}(x,y,t) = 1$ and $N_{d_{\sqsubseteq 0}}(x,y,t) = 0$ if x is a prefix of y and $t > 0$, $M_{d_{\sqsubseteq 1}}(x,y,t) = 1 - 2^{-l(x \sqcap y)}$ and $N_{d_{\sqsubseteq 0}}(x,y,t) = 2^{-l(x \sqcap y)}$

if x is not a prefix of y and $t \in (0,1)$,

$M_{d_{\sqsubseteq 1}}(x,y,t) = 1$ and $N_{d_{\sqsubseteq 0}}(x,y,t) = 0$ if x is not a prefix of y and $t > 1$.

Proposition 4.5.2 allows us to apply any of the *Proposition* 4.5.1 and *Theorem* 4.3.1 to the complexity analysis of quicksort algorithm, to show, in direct way, the existence and uniqueness of solution for the following recurrence equation:

$$T(1) = 0 \text{ and } T(n) = \frac{2(n-1)}{n} + \frac{n+1}{n} T(n-1), \; n \geq 2$$

The average case analysis of Quicksort is discussed in [14] (see also [6]), where the above recurrence equation is obtained

Consider as an alphabet Σ the set of non-negative real numbers, that is, $\Sigma = [0,1)$. We associate to T the functional $\Phi : \Sigma^{\infty} \rightarrow \Sigma^{\infty}$ given by $\left(\Phi(x)\right)_1 = T(1)$ and $\left(\Phi(x)\right)_n = \frac{2(n-1)}{n} + \frac{n+1}{n} x_{(n-1)}, \; n \geq 2$

If $x \in \Sigma^\infty$ has length $n < 1$, we write $x = x_1x_2x_3 \dots x_n$, and if x is an infinite word we write $x = x_1x_2x_3 \dots x_n \dots$ Next we show that Φ is a B −contraction on the G −bicomplete non-Archimedean intuitionistic fuzzy quasi-metric space $(\Sigma^\infty, M_{d_\sqsubseteq}, N_{d_\sqsubseteq}, \wedge, \vee)$ with contraction constant $\frac{1}{2}$.

To this end, we first note that, by construction,

we have $l(\Phi(x)) = l(x) + 1$

for all $x \in \Sigma^\infty$ (in particular $l(\Phi(x)) = 1$ whenever $l(x) = 1$).

Furthermore, it is clear that $x \sqsubseteq y \Leftrightarrow \Phi(x) \sqsubseteq \Phi(y)$,

and consequently $\Phi(x \sqcap y) \sqsubseteq \Phi(x) \sqcap \Phi(y)$, for all $x, y \in \Sigma^\infty$.

Hence $l(\Phi(x \sqcap y)) \sqsubseteq l(\Phi(x) \sqcap \Phi(y))$, for all $x, y \in \Sigma^\infty$.

From the preceding observations we deduce that for all $x, y \in X$, if x is a prefix of y, then $M_{d_\sqsubseteq}\left(\Phi(x), \Phi(y), \frac{t}{2}\right) = M_{d_\sqsubseteq}(x, y, t) = 1$

and $N_{d_\sqsubseteq}\left(\Phi(x), \Phi(y), \frac{t}{2}\right) = N_{d_\sqsubseteq}(x, y, t) = 0.$

and if x is not a prefix of y, then for all $t > 0$,

$$\begin{aligned}
M_{d_\sqsubseteq}\left(\Phi(x), \Phi(y), \tfrac{t}{2}\right) &= \frac{\frac{t}{2}}{\left(\frac{t}{2}\right) + 2^{-l(\Phi(x) \sqcap \Phi(y))}} \\
&\geq \frac{\frac{t}{2}}{\left(\frac{t}{2}\right) + 2^{-l(\Phi(x \sqcap y))}} \\
&\geq \frac{\frac{t}{2}}{\left(\frac{t}{2}\right) + 2^{-(l(x \sqcap y)+1)}} \\
&\geq \frac{t}{t + 2^{-(l(x \sqcap y))}} \\
&\geq M_{d_\sqsubseteq}(x, y, t)
\end{aligned}$$

and

$$\begin{aligned}
N_{d_\sqsubseteq}\left(\Phi(x), \Phi(y), \tfrac{t}{2}\right) &= \frac{2^{-l(\Phi(x) \sqcap \Phi(y))}}{\left(\frac{t}{2}\right) + 2^{-l(\Phi(x) \sqcap \Phi(y))}} \\
&\leq \frac{2^{-l(\Phi(x \sqcap y))}}{\left(\frac{t}{2}\right) + 2^{-l(\Phi(x \sqcap y))}} \\
&\leq \frac{2^{-(l(x \sqcap y)+1)}}{\left(\frac{t}{2}\right) + 2^{-(l(x \sqcap y)+1)}} \\
&\leq \frac{2^{-(l(x \sqcap y))}}{t + 2^{-(l(x \sqcap y))}} \\
&\leq N_{d_\sqsubseteq}(x, y, t)
\end{aligned}$$

Therefore, Φ is a B −contraction on $(\Sigma^\infty, M_{d_\sqsubseteq}, N_{d_\sqsubseteq}, \wedge, \vee)$ with contraction constant $\frac{1}{2}$. So, by *Theorem* 1.3.1, Φ has a unique fixed point $z = z_1z_2z_3 \dots$, which is obviously the unique solution to the recurrence equation T, that is, $z_1 = 0$ and

$$z_n = \frac{2(n-1)}{n} + \frac{n+1}{n} z_{(n-1)}, \ n \geq 2 .$$

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Research Article

THE SUM OF i^{th} SMALLEST PARTS OF $r-partitions$ of n

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ABSTRACT

George E Andrews [1] derived generating function for the number of smallest parts of *partitions* of positive integer *n*. Hanuma Reddy [2] defined i^{th} smallest part and derived a relation between the i^{th} smallest and i^{th} greatest parts of *partitions* of *n* in general form. Here we derive generating function for the sum of the i^{th} smallest parts of $r-partitions$ of *n*.

INTRODUCTION

A *partition* of a positive integer n is a finite non increasing sequence of positive integers $\lambda_1, \lambda_2, ..., \lambda_r$ such that $\sum_{i=1}^{r} \lambda_i = n$ and is denoted by $n = (\lambda_1, \lambda_2, ..., \lambda_r)$, $n = \lambda_1 + \lambda_2 + \lambda_3 + ...\lambda_r$ or $\lambda = \left(\lambda_1^{f_1}, \lambda_2^{f_2}, \lambda_3^{f_3}, ...\right)$ when λ_1 repeats f_1 times, λ_2 repeats f_2 times and so on. The λ_i are called the parts of the *partition*. In what follows λ stands for a *partition* of n, $\lambda = (\lambda_1, \lambda_2, ..., \lambda_r), \lambda_1 \geq \lambda_2 \geq ... \geq \lambda_r$. The set of all *partitions* of n is represented by $\xi(n)$ by and its cardinolity $p(n)$.

If $1 \leq r \leq n$ then $\xi_r(n)$ is the set of *partition*s of n with r parts and its cardinality is denoted by $p_r(n)$. A *partition* of *n* with exactly *r* parts is called $r-partition$ of *n*. We define

$$p_r(n) = \begin{cases} 0 & \text{if } \quad r=0 \quad or \quad r>n \\ \text{number of } r-partitions \text{ of } n & \text{if } \quad 0<r\leq n \end{cases}$$

$spt(n)$ denotes the number of smallest parts including repetitions in all *partitions* of n. $spt_i(n)$ denotes the number of i^{th} smallest parts including repetitions in all *partitions* of n. $r-spt_i(n)$ denotes the number of i^{th} smallest parts in all $r-partitions$ of *n*. The number of $partitions$ of n with least part greater than or equal to k is represented by $p(k,n)$.

Existing generating functions are given below.

Function	Generating function
$p_r(n)$	$\dfrac{q^r}{(q)_r}$
$p_r(n-k)$	$\dfrac{q^{r+k}}{(q)_r}$
number of divisors	$\sum_{n=1}^{\infty} \dfrac{q^n}{(1-q^n)}$
sum of divisors	

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$$\sum_{n=1}^{\infty}\frac{n.q^n}{\left(1-q^n\right)} \qquad (1.1.1)$$

where $(q)_k=\prod_{n=1}^{k}\left(1-q^n\right)$ for $k>0$, $(q)_k=1$ for $k=0$ and $(q)_k=0$ for $k<0$.

and $(a)_n=(a;q)_n=(1-a)(1-aq)\left(1-aq^2\right)\ldots\left(1-aq^{n-1}\right)$

(1.2) Theorem: The generating function for the sum of smallest parts of the *partitions* of n is

$$\sum_{n=1}^{\infty} sum\, spt(n)q^n = \frac{1}{(q)_\infty}\sum_{n=1}^{\infty}\frac{nq^n(q)_{n-1}}{\left(1-q^n\right)}$$

Proof : From [4] we have the sum of smallest parts $sum\ spt(n)$ of the *partitions* of a positive integer n is

$$sum\ spt(n)=\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}k\ \ p(k,n-tk)+\sigma(n)$$

where $\sigma(n)$ is sum of positive divisors of n.

$$=\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty}k\ \ p_r(k,n-tk)$$

First replace $k+1$ by k, then replace n by $n-tk$ in [3]

$$=\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty}k\ p_r\big(n-tk-r(k-1)\big)+\sigma(n)$$

Hence the generating function for the sum of smallest parts of the *partitions* of a positive integer n is

$$\sum_{n=1}^{\infty} sum\,spt(n)q^n=\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty}\frac{kq^{r+tk+r(k-1)}}{(q)_r}+\sum_{k=1}^{\infty}\frac{kq^k}{1-q^k}$$

from (1.1.1)

$$=\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty}\frac{kq^{tk+rk}}{(q)_r}+\sum_{k=1}^{\infty}\frac{kq^k}{1-q^k}$$

$$=\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}kq^{tk}\left[\sum_{r=1}^{\infty}\frac{\left(q^k\right)^r}{(q)_r}\right]+\sum_{k=1}^{\infty}\frac{kq^k}{1-q^k}$$

$$=\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}kq^{tk}\left[1+\sum_{r=1}^{\infty}\frac{\left(q^k\right)^r}{(q)_r}-1\right]+\sum_{k=1}^{\infty}\frac{kq^k}{1-q^k}$$

$$=\sum_{k=1}^{\infty}\frac{kq^k}{\left(1-q^k\right)}\prod_{r=0}^{\infty}\left(\frac{1}{1-q^rq^k}\right)$$

$$=\sum_{k=1}^{\infty}\frac{kq^k}{\left(1-q^k\right)}\prod_{r=0}^{\infty}\left(\frac{1}{1-q^{r+k}}\right)$$

$$=\frac{1}{(q)_\infty}\sum_{k=1}^{\infty}\frac{kq^k(q)_{k-1}}{\left(1-q^k\right)}$$

$$=\frac{1}{(q)_\infty}\sum_{n=1}^{\infty}\frac{nq^n(q)_{n-1}}{\left(1-q^n\right)}$$

$$\sum_{n=1}^{\infty} sum\,spt(n)q^n=\frac{1}{(q)_\infty}\sum_{n=1}^{\infty}\frac{nq^n(q)_{n-1}}{\left(1-q^n\right)} \blacksquare$$

(1.3) Theorem: The sum $r-spt(n)$ of i^{th} smallest parts of $r-partitions$ of n such that i^{th} smallest part is first part $\left(i.e\ \lambda_1 \text{ as } i^{th}\text{smallest part}\right)$ is

$$\sum_{n=1}^{\infty} sum\left(r-spt_i(n)\right)q^n$$

$$=\left\{\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)}\right.$$

$$+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)}$$

$$\left.+\ldots+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)^2}\right\}\ \text{for } i=r \qquad (1.3.1)$$

Proof: Let $n=(\lambda_1,\lambda_2,\ldots,\lambda_r)=\left(\mu^\alpha\right)$ be any $r-partition$ of n with all parts equal.

We know that the sum of the smallest parts of $r-partitions$ of n such that the smallest part is the first part $\left(i.e\ \lambda_1 \text{ as smallest part}\right)$ and having k as smallest part is $k\beta$

$$\text{where } \beta=\begin{cases}1 & \text{if } \dfrac{n}{r}=k\\ 0 & \text{otherwise}\end{cases} \qquad (1.3.2)$$

The generating function for the sum of smallest parts of $r-partitions$ of n such that smallest part as the first part $\left(i.e\ \lambda_1 \text{ as smallest part}\right)$

$$\sum_{n=1}^{\infty}\left(r-spt_1(n)\right)q^n=\sum_{k=1}^{\infty}k.q^{kr}==\frac{q^r}{\left(1-q^r\right)^2}\quad \text{for } r=1$$

Let $n=(\lambda_1,\lambda_2,\ldots,\lambda_r)=\left(\mu_1^{\alpha_1},\mu_2^{\alpha_2}\right)$ be any $r-partition$ of n with two distinct parts.

Subtracting μ_2 from each λ_i for $i=1$ to r, we get $n_1=\left(\mu_1^{(1)}\right)^{\alpha_1}$ where $n_1=n-r\mu_2$, $r_1=r-\alpha_2$ and $\mu_1^{(1)}=\mu_1-\mu_2$

The sum of the smallest parts of $r_1-partitions$ of n_1 such that smallest part is the first part and having k as the smallest part is $k\beta_1$.

$$\text{where } \beta_1=\begin{cases}1 & \text{if } r_1 \mid n_1\\ 0 & \text{otherwise}\end{cases} \text{ and } \qquad n=n_1+r\mu_2,$$

$\mu_1=k-\mu_2$.

Now the sum of second smallest parts of $r-partitions$ of n such that the second smallest part is the first part and having k as the smallest part is $k\beta_1$.

$$\text{where } \beta_1=\begin{cases}1 & \text{if } \dfrac{n-r\mu_2}{r_1}=k-\mu_2\\ 0 & \text{otherwise}\end{cases}$$

Therefore the generating function for the sum of the second smallest parts of $r-partitions$ of n such that the second smallest part as the first part $\left(i.e\ \lambda_1 \text{ as second smallest part}\right)$ is

$$\sum_{n=1}^{\infty} sum\left(r-spt_2\left(n\right)\right)q^n=\sum_{\mu_2=1}^{\infty}\sum_{r_1=1}^{r-1}\sum_{k-\mu_2=1}^{\infty} kq^{\mu_2 r+\left(k-\mu_2\right)r_1}$$

$$=\sum_{\mu_2=1}^{\infty}\sum_{r_1=1}^{r-1}\sum_{\mu_1=1}^{\infty}\left(\mu_1+\mu_2\right)q^{\mu_1 r_1+\mu_2 r}$$

$$=\left[\sum_{\mu_2=1}^{\infty}\mu_2 q^{\mu_2 r}\sum_{\mu_1=1}^{\infty}\sum_{r_1=1}^{r-1}q^{\mu_1 r_1}+\sum_{\mu_2=1}^{\infty}q^{\mu_2 r}\sum_{\mu_1=1}^{\infty}\sum_{r_1=1}^{r-1}\mu_1 q^{\mu_1 r_1}\right]$$

$$=\left[\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\right]$$

for $r=2$ (1.3.4)

Continuing this process, we get the generating function for the sum of i^{th} smallest parts of $r-partitions$ of n such that i^{th} smallest part as first part $\left(i.e\ \lambda_1 \text{ as } i^{th} \text{smallest part}\right)$ is

$$\sum_{n=1}^{\infty} sum\left(r-spt_i\left(n\right)\right)q^n$$

$$=\left\{\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)}\right.$$

$$+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)}$$

$$\left.+\ldots\ldots+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)^2}\right\} \text{ for } i=r \qquad \blacksquare$$

(1.4) Theorem: The generating function for the sum of i^{th} smallest parts of $r-partitions$ of n is

$$\sum_{n=1}^{\infty} sum\left(r-spt_i\left(n\right)\right)q^n$$

$$=\left\{\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)}\left(\sum_{r_i=1}^{r_{i-1}}\frac{1}{(q)_{r_i}}+1\right)\right.$$

$$+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)}\left(\sum_{r_i=1}^{r_{i-1}}\frac{1}{(q)_{r_i}}+1\right)$$

$$\left.+\ldots+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)^2}\left(\sum_{r_i=1}^{r_{i-1}}\frac{1}{(q)_{r_i}}+1\right)\right\}$$

Proof: The sum of smallest parts of $r-partitions$ of n having k as a smallest part is

$$k\sum_{i=0}^{\infty}p_{r-1-i}\left[n-\left(k-1\right)r-1-i\right]+k\beta$$

$$\text{where } \beta=\begin{cases}1 & \text{if } \dfrac{n}{r}=k\\ 0 & \text{otherwise}\end{cases}$$

The generating function for the sum of the smallest parts of $r-partitions$ of n is

$$\sum_{n=1}^{\infty} sum\left(r-spt\left(n\right)\right)q^n=\sum_{k=1}^{\infty}\sum_{i=0}^{\infty}\frac{kq^{r-1-i+\left(k-1\right)r+1+i}}{\left(q\right)_{r-1-i}}+\frac{q^r}{\left(1-q^r\right)^2}$$

$$=\sum_{k=1}^{\infty}\sum_{i=0}^{\infty}\frac{kq^{kr}}{\left(q\right)_{r-1-i}}+\frac{q^r}{\left(1-q^r\right)^2}$$

$$=\sum_{i=0}^{\infty}\frac{\left(q^r+2q^{2r}+3q^{3r}+\ldots\right)}{\left(q\right)_{r-1-i}}+\frac{q^r}{\left(1-q^r\right)^2}$$

$$=\frac{q^r}{\left(1-q^r\right)^2}\sum_{i=1}^{r-1}\frac{1}{\left(q\right)_i}+\frac{q^r}{\left(1-q^r\right)^2}$$

$$=\frac{q^r}{\left(1-q^r\right)^2}\left(\sum_{i=1}^{r-1}\frac{1}{\left(q\right)_i}+1\right)$$

$$=\frac{q^r}{\left(1-q^r\right)^2}\left(\sum_{r_1=1}^{r-1}\frac{1}{\left(q\right)_{r_1}}+1\right)$$

From theorems (3.3.1) and (3.3.4) in [5], the sum of second smallest parts (without λ_1) of $r-partitions$ of n with least part k is

$$\sum_{i=0}^{\infty}\sum_{r_1=1}^{r-1}\sum_{\mu_l=1}^{\infty}\sum_{k=1}^{\infty}k.p_{r_1-1-i}\left(n-\mu_l r-\left(k-\mu_l-1\right)r_1-1-i\right)$$

The generating function for the sum of second smallest parts (without λ_1) of $r-partitions$ of n is

$$\sum_{\mu_l=1}^{\infty}\sum_{i=0}^{\infty}\sum_{r_1=1}^{r-1}\sum_{k=1}^{\infty}\frac{kq^{r_1-1-i+\mu_l r+\left(k-\mu_l-1\right)r_1+1+i}}{(q)_{r_1-1-i}}$$

$$=\sum_{\mu_l=1}^{\infty}\sum_{i=0}^{\infty}\sum_{r_1=1}^{r-1}\sum_{k=1}^{\infty}\frac{kq^{\mu_l r+kr_1-\mu_l r_1}}{(q)_{r_1-1-i}}$$

$$=\sum_{\mu_l=1}^{\infty}\sum_{i=0}^{\infty}\sum_{r_1=1}^{r-1}\sum_{k=\mu_l+1}^{\infty}\frac{kq^{\mu_l(r-r_1)+kr_1}}{(q)_{r_1-1-i}}$$

$$=\sum_{\mu_l=1}^{\infty}\sum_{i=0}^{\infty}\sum_{r_1=1}^{r-1}\sum_{\mu_{l-1}=1}^{\infty}\frac{\left(\mu_l+\mu_{l-1}\right)q^{\mu_l(r-r_1)+(\mu_l+\mu_{l-1})r_1}}{(q)_{r_1-1-i}}$$

where $k=\mu_l+\mu_{l-1}$

$$=\sum_{\mu_l=1}^{\infty}\sum_{i=0}^{\infty}\sum_{r_1=1}^{r-1}\sum_{\mu_{l-1}=1}^{\infty}\frac{\left(\mu_l+\mu_{l-1}\right)q^{\mu_l r+\mu_{l-1}r_1}}{(q)_{r_1-1-i}}$$

$$=\sum_{\mu_l=1}^{\infty}\sum_{i=0}^{\infty}\sum_{r_1=1}^{r-1}\sum_{\mu_{l-1}=1}^{\infty}\frac{\mu_l q^{\mu_l r}q^{\mu_{l-1}r_1}}{(q)_{r_1-1-i}}+\sum_{\mu_l=1}^{\infty}\sum_{i=0}^{\infty}\sum_{r_1=1}^{r-1}\sum_{\mu_{l-1}=1}^{\infty}\frac{\mu_{l-1}q^{\mu_l r}q^{\mu_{l-1}r_1}}{(q)_{r_1-1-i}}$$

$$=\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{1}{(q)_{r_2}}+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\sum_{r_2=1}^{r_1-1}\frac{1}{(q)_{r_2}}$$

$$=\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{1}{(q)_{r_2}}+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\sum_{r_2=1}^{r_1-1}\frac{1}{(q)_{r_2}}$$

The generating function for the sum of second smallest parts of $r-partitions$ of n are equal to

$$\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\quad \text{for } r=2$$

Therefore the generating function for the sum of second smallest parts of $r-partitions$ of n is

$$\sum_{n=1}^{\infty}sum\left(r-spt_2(n)\right)q^n=$$

$$\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\left(\sum_{r_2=1}^{r_1-1}\frac{1}{(q)_{r_2}}+1\right)+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\left(\sum_{r_2=1}^{r_1-1}\frac{1}{(q)_{r_2}}+1\right)$$

Similarly the generating function for the sum of third smallest parts of $r-partitions$ of n is

$$\sum_{n=1}^{\infty}sum\left(r-spt_3(n)\right)q^n$$

$$=\left\{\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\left(\sum_{r_3=1}^{r_2-1}\frac{1}{(q)_{r_3}}+1\right)\right.$$

$$+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\left(\sum_{r_3=1}^{r_2-1}\frac{1}{(q)_{r_3}}+1\right)$$

$$\left.+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)^2}\left(\sum_{r_3=1}^{r_2-1}\frac{1}{(q)_{r_3}}+1\right)\right\}$$

By induction, we get the generating function for the sum of the i^{th} smallest parts of $r-partitions$ of n which is given by

$$\sum_{n=1}^{\infty}sum\left(r-spt_i(n)\right)q^n$$

$$=\left\{\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)}\left(\sum_{r_i=1}^{r_{i-1}}\frac{1}{(q)_{r_i}}+1\right)\right.$$

$$+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)}\left(\sum_{r_i=1}^{r_{i-1}}\frac{1}{(q)_{r_i}}+1\right)$$

$$\left.+\ldots+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)^2}\left(\sum_{r_i=1}^{r_{i-1}}\frac{1}{(q)_{r_i}}+1\right)\right\}$$

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The number of i^{th} smallest parts of $r-partitions$ of n

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Abstract: George E Andrews [1] derived generating function for the number of smallest parts of *partitions* of positive integer *n*. Hanuma Reddy [2] defined i^{th} smallest part and derived a relation between the i^{th} smallest and i^{th} greatest parts of *partitions* of *n* in general form. Here we derive generating function for the number of the i^{th} smallest parts of $r-partitions$ of *n.*

Keywords: *partitions*, *r-partitions*, smallest parts of *partition* and i^{th} smallest parts of *partition* of positive integer *n.*

Subject classification: 11P81 Elementary theory of *partitions*.

Introduction:

A *partition* of a positive integer n is a finite non increasing sequence of positive integers $\lambda_1,\lambda_2,...,\lambda_r$ such that $\sum_{i=1}^{r}\lambda_i=n$ and is denoted by $n=(\lambda_1,\lambda_2,...,\lambda_r)$, $n=\lambda_1+\lambda_2+\lambda_3+...\lambda_r$ or $\lambda=(\lambda_1^{f_1},\lambda_2^{f_2},\lambda_3^{f_3},...)$ when λ_1 repeats f_1 times, λ_2 repeats f_2 times and so on. The λ_i are called the parts of the *partition*. In what follows λ stands for a *partition*of n, $\lambda=(\lambda_1,\lambda_2,...,\lambda_r),\lambda_1\ge\lambda_2\ge...\ge\lambda_r$. The set of all *partitions* of n is represented by $\xi(n)$ by and its cardinolity $p(n)$.

If $1\le r\le n$ then $\xi_r(n)$ is the set of *partitions* of *n* with *r* parts and its cardinality is denoted by $p_r(n)$. A *partition* of *n* with exactly *r* parts is called $r-partition$ of *n*. We define

$$p_r(n)=\begin{cases}0 & \text{if } \quad r=0 \quad or \quad r>n\\ \text{number of } r-partitions \text{ of } n & \text{if } \quad 0<r\le n\end{cases}$$

$spt(n)$ denotes the number of smallest parts including repetitions in all *partitions* of n. $spt_i(n)$ denotes the number of i^{th} smallest parts including repetitions in all *partitions* of n. $r-spt_i(n)$ denotes the number of i^{th} smallest parts in all $r-partitions$ of *n*.The number of *partitions* of n with least part greater than or equal to k is represented by $p(k,n)$.

1.1 Existing generating functions are given below.

Function	Generating function	
$p_r(n)$	$\dfrac{q^r}{(q)_r}$	
$p_r(n-k)$	$\dfrac{q^{r+k}}{(q)_r}$	
number of divisors	$\displaystyle\sum_{n=1}^{\infty}\frac{q^n}{(1-q^n)}$	
sum of divisors	$\displaystyle\sum_{n=1}^{\infty}\frac{n.q^n}{(1-q^n)}$	(1.1.1)

where $(q)_k=\prod_{n=1}^{k}(1-q^n)$ for $k>0$, $(q)_k=1$ for $k=0$ and $(q)_k=0$ for $k<0$.

and $(a)_n=(a;q)_n=(1-a)(1-aq)(1-aq^2)\dots(1-aq^{n-1})$

1.2Theorem: If $k\in N$ and $1\le k\le\left[\dfrac{n}{r}\right]$, then the number $f_r^{\,i}(k,n)$ of $r-partitions$ of n with k as i^{th} smallest part is

i) $f_r(k,n)=p_{r-1}\left[n-(k-1)r-1\right]+\beta$ for $i=1$

$$\text{where } \beta=\begin{cases}1 & \text{if } \dfrac{n}{r}=k\\ 0 & \text{otherwise}\end{cases}$$

$ii)$ If $i>1$

$$f_r^{\,i}(k,n)=\sum_{r_1=1}^{r-1}\sum_{\mu_l=1}^{\infty}\dots\sum_{r_{i-2}=1}^{r_{i-3}-1}\sum_{\mu_{l-i+2}=1}^{\infty}p_{r_{i-1}-1}\left[\begin{array}{l}(n-r\mu_l-r_1\mu_{l-1}-\dots-r_{i-2}\mu_{l-i+2})\\ \qquad\qquad -(k-1)(r-\alpha_l\dots-\alpha_{l-i+2})-1\end{array}\right]$$

$$+\sum_{r_1=1}^{r-1}\sum_{\mu_l=1}^{\infty}\dots\sum_{r_{i-2}=1}^{r_{i-3}-1}\sum_{\mu_{l-i+2}=1}^{\infty}\beta_{i-1} \qquad \text{for } i>1$$

$$\text{where } \beta_{i-1}=\begin{cases}1 & \text{if } \dfrac{n-r\mu_l-r_1\mu_{l-1}-\dots-r_{i-2}\mu_{l-i+2}}{r_{i-1}}=k-\mu_l\dots-\mu_{l-i+2}\\ 0 & \text{otherwise}\end{cases}$$

Proof :

(i) For $i=1$

Let $n=(\lambda_1,\lambda_2,\ldots,\lambda_r)=\left(\mu_1^{\alpha_1},\mu_2^{\alpha_2},,\ldots,\mu_{l-1}^{\alpha_{l-1}},k^{\alpha_l}\right)$ be any $r-partition$ of n with l distinct parts.

Put $t=1$ in theorem 1.2 in [3], we get the number of $r-partitions$ of n with k as smallest part is

$$f_r(k,n)=p_{r-1}(k,n-k)+\beta$$

$$\text{where } \beta=\begin{cases}1 & \text{if } \dfrac{n}{r}=k\\ 0 & \text{otherwise}\end{cases}$$

First replace $k+1$ by k, r by $r-1,$ then replace n by $n-k$ in theorem 1.3 in [3], we get

$$=p_{r-1}\left[n-(k-1)r-1\right]+\beta \quad (1.2.1)$$

(ii) For $i>1$

Let $n=(\lambda_1,\lambda_2,\ldots,\lambda_r)$

$$=\left(\mu_1^{\alpha_1},\ldots,\mu_{l-i}^{\alpha_{l-i}},\mu_{l-i+1}^{\alpha_{l-i+1}},\mu_{l-i+2}^{\alpha_{l-i+2}},\ldots,\mu_{l-1}^{\alpha_{l-1}},\mu_l^{\alpha_l}\right) \text{ (1.2.2)}$$

be any $r-partition$ of n with l distinct parts. Subtracting μ_l from λ_i for $i=1$ to r, we get

$$n_1=(\lambda_1^{(1)},\lambda_2^{(1)},\ldots,\lambda_{r_1}^{(1)})=\left(\left(\mu_1^{(1)}\right)^{\alpha_1},\ldots,\left(\mu_{l-i}^{(1)}\right)^{\alpha_{l-i}},\left(\mu_{l-i+1}^{(1)}\right)^{\alpha_{l-i+1}},\left(\mu_{l-i+2}^{(1)}\right)^{\alpha_{l-i+2}},\ldots,\left(\mu_{l-1}^{(1)}\right)^{\alpha_{l-1}}\right)$$

$$\text{where } n_1=n-r\mu_l, r_1=r-\alpha_l \text{ and } \mu_\varphi^{(1)}=\mu_\varphi-\mu_l\forall\varphi \qquad (1.2.3)$$

From (1.2.1) we have the number of $r_1-partitions$ of n_1 having smallest element k is

$$p_{r_1-1}\left[n_1-(k-1)r_1-1\right]+\beta_1$$

$$\text{where } \beta_1=\begin{cases}1 & \text{if } \dfrac{n_1}{r_1}=k\\ 0 & \text{otherwise}\end{cases}$$

$$=p_{r_1-1}\left[(n-r\mu_l)-(k-1)r_1-1\right]+\beta_1 \text{(1.2.4)}$$

$$\text{where } \beta_1=\begin{cases}1 & \text{if } \dfrac{n-r\mu_l}{r_1}=k-\mu_l\\ 0 & \text{otherwise}\end{cases}$$

In (1.2.2), the part μ_l may vary from 1 to $\mu_{l-1}-1$ and r_1 may vary from 1 to $r-1$(if $\mu_l=\mu_{l-1}$ or $r_1=r,$ the $partition$(1.2.2) does not have l distinct parts.

It contradicts our assumption for $\mu_l>\mu_{l-1}$.)

Therefore the number of $r-partitions$ of n with second smallest part k is $f_r^{\,2}(k,n)$

$$f_r^{\,2}(k,n)=\sum_{r_1=1}^{r-1}\sum_{\mu_l=1}^{\infty}p_{r_1-1}\left[(n-r\mu_l)-(k-1)r_1-1\right]+\sum_{r_1=1}^{r-1}\sum_{\mu_l=1}^{\infty}\beta_1 \qquad (1.2.5)$$

Continuing this process in(1.2.3), we get

$$n_h=(\lambda_1^{(h)},\lambda_2^{(h)},\ldots,\lambda_{r_h}^{(h)})=\left(\left(\mu_1^{(h)}\right)^{\alpha_1},\left(\mu_2^{(h)}\right)^{\alpha_2},\ldots,\left(\mu_{l-h-1}^{(h)}\right)^{\alpha_{l-h-1}},\left(\mu_{l-h}^{(h)}\right)^{\alpha_{l-h}}\right)$$

where $n_0=n, n_h=n_{h-1}-r_{h-1}\mu_{l-h+1}, r_0=r,\ r_h=r_{h-1}-\alpha_{l-h+1}$ and $\mu_\varphi^{(h)}=\mu_\varphi^{(h-1)}-\mu_{l-h+1}\forall\varphi$

From (1.2.1), we have the number of $r_h-partition$ of n_h having smallest part k is

$$p_{r_h-1}\left[n_h-(k-1)r_h-1\right]+\beta_h$$

where $\beta_h=\begin{cases}1 & \text{if } r_h\mid n_h\\ 0 & \text{otherwise}\end{cases}$

Hence the number $f_r^i(k,n)$ of $r_{i-1}-partitions$ of n_{i-1} with i^{th} smallest part k as

$$f_r^i(k,n)=p_{r_{i-1}-1}\left[n_{i-1}-(k-1)r_{i-1}-1\right]+\beta_{i-1}$$

where $\beta_{i-1}=\begin{cases}1 & \text{if } r_{i-1}\mid n_{i-1}\\ 0 & \text{otherwise}\end{cases}$

$$=\sum_{r_1=1}^{r-1}\sum_{\mu_l=1}^{\infty}\cdots\sum_{r_{i-2}=1}^{r_{i-3}-1}\sum_{\mu_{l-i+2}=1}^{\infty}p_{r_{i-1}-1}\left[\begin{aligned}&(n-r\mu_l-r_1\mu_{l-1}-\ldots-r_{i-2}\mu_{l-i+2})\\ &\qquad -(k-1)(r-\alpha_l\ldots-\alpha_{l-i+2})-1\end{aligned}\right]$$

$$+\sum_{r_1=1}^{r-1}\sum_{\mu_l=1}^{\infty}\cdots\sum_{r_{i-2}=1}^{r_{i-3}-1}\sum_{\mu_{l-i+2}=1}^{\infty}\beta_{i-1}$$

where $\beta_{i-1}=\begin{cases}1 & \text{if } \dfrac{n-r\mu_l-r_1\mu_{l-1}-\ldots-r_{i-2}\mu_{l-i+2}}{r_{i-1}}=k-\mu_l\ldots-\mu_{l-i+2}\\ 0 & \text{otherwise}\end{cases}$

This completes the proof ■

1.3Theorem: The generating function for the number of i^{th} smallest parts of $r-partitions$ of n such that i^{th} smallest part as first part $\left(i.e\ \lambda_1 \text{ as } i^{th}\text{smallest part}\right)$ is

$$\sum_{n=1}^{\infty}\left(r-spt_i(n)\right)q^n=\frac{q^r}{(1-q^r)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{(1-q^{r_1})}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{(1-q^{r_2})}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{(1-q^{r_{i-1}})}\quad \text{for } i=r \quad (1.3.1)$$

Proof: Let $n=(\lambda_1,\lambda_2,\ldots,\lambda_r)=\left(\mu^r\right)$ be any $r-partition$ of n with all equal parts.

We know that β is the number of smallest parts of $r-partitions$ of n such that smallest part is the first part which is k $\left(i.e\ \lambda_1 \text{ as smallest part}\right)$.

where $\beta=\begin{cases}1 & \text{if } \dfrac{n}{r}=k\\ 0 & \text{otherwise}\end{cases}$ (1.3.2)

The generating function for the number of smallest parts of $r-partitions$ of n such that smallest part is the first part $\left(i.e\ \lambda_1 \text{ as smallest part}\right)$ is

$$\sum_{n=1}^{\infty}\left(r-spt_1\left(n\right)\right)q^n=\sum_{k=1}^{\infty}q^{kr}=\frac{q^r}{\left(1-q^r\right)} \text{ for } r=1 \qquad (1.3.3)$$

Let $n=(\lambda_1,\lambda_2,\ldots,\lambda_r)=\left(\mu_1{}^{\alpha_1},\mu_2{}^{\alpha_2}\right)$ be any $r-partition$ of n with two distinct parts.

Subtracting μ_2 from each λ_i for $i=1$ to r, we get

$$n_1=\left(\mu_1^{(1)}\right)^{\alpha_1} \quad \text{where} \quad n_1=n-r\mu_2, \quad r_1=r-\alpha_2 \quad \text{and} \quad \mu_1^{(1)}=\mu_1-\mu_2$$

The number of smallest parts of $r_1-partitions$ of n_1 such that the smallest part is the first part and having k as a smallest part is β_1

$$\text{where } \beta_1=\begin{cases}1 & \text{if } r_1 \mid n_1\\ 0 & \text{otherwise}\end{cases}$$

Since $n=n_1+r\mu_2$ and $\mu_1=k-\mu_2$, the number of second smallest parts of $r-partitions$ of n such that second smallest part is the first part and having k as a smallest part is β_1

$$\text{where } \beta_1=\begin{cases}1 & \text{if } \dfrac{n-r\mu_2}{r_1}=k-\mu_2\\ 0 & \text{otherwise}\end{cases}$$

Therefore the generating function for the number of second smallest parts of $r-partitions$ of n such that second smallest part is the first part $\left(i.e\ \lambda_1 \text{ as second smallest part}\right)$ is

$$\sum_{n=1}^{\infty}\left(r-spt_2\left(n\right)\right)q^n=\sum_{\mu_2=1}^{\infty}\sum_{r_1=1}^{r-1}\sum_{k-\mu_2=1}^{\infty}q^{\mu_2 r+\left(k-\mu_2\right)r_1}$$

$$=\sum_{\mu_2=1}^{\infty}\sum_{r_1=1}^{r-1}\sum_{\mu_1=1}^{\infty}q^{\mu_1 r_1+\mu_2 r}$$

$$=\sum_{\mu_2=1}^{\infty}q^{\mu_2 r}\sum_{\mu_1=1}^{\infty}\sum_{r_1=1}^{r-1}q^{\mu_1 r_1}$$

$$=\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)} \text{ for } r=2 \quad (1.3.4)$$

Continuing this process, we getthe generating function for the number of i^{th} smallest parts of $r-partitions$ of n such that i^{th} smallest part as first part $\left(i.e\ \lambda_1 \text{ as } i^{th}\text{smallest part}\right)$ is

$$\sum_{n=1}^{\infty}\left(r-spt_i\left(n\right)\right)q^n=\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)} \text{ for } i=r \quad \blacksquare$$

1.4Theorem: The number of smallest parts of $r-partitions$ of n having k as a smallest part is

$$\sum_{i=0}^{\infty} p_{r-1-i}\left[n-(k-1)r-1-i\right]+\beta$$

where $\beta=\begin{cases}1 & \text{if} \quad r\,|\,n \\ 0 & \text{otherwise}\end{cases}$

Proof: From (1.2.10), the number of $r-partitions$ of n with the smallest part k is

$$f_r(k,n)=p_{r-1}\left[n-(k-1)r-1\right]+\beta$$

Fix $k\in\{1,2,...,n\}$.For $1\le i\le r$ the number of $r-partitions$ of n with the $(r-i)$ smallest parts each being k is the number of $r-partitions$ of $n-(r-i)k$. Summing over $i=1$ to r we get the total number of $r-partitions$ of n with k as the smallest parts .

This number $\sum_{i=0}^{\infty} p_{r-1-i}\left[n-(k-1)r-1-i\right]+\beta$ where $\beta=\begin{cases}1 & \text{if} \quad r\,|\,n \\ 0 & \text{otherwise}\end{cases}$ ■

1.5Theorem: The generating function for the number of i^{th} smallest parts of $r-partitions$ of n is

$$\sum_{n=1}^{\infty} r-spt_i(n)q^n=\frac{q^n}{(1-q^n)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{(1-q^{r_1})}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{(1-q^{r_2})}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{(1-q^{r_{i-1}})}\left(\sum_{r_i=1}^{r_{i-1}-1}\frac{1}{(q)_{r_i}}+1\right)$$

Proof: From theorem1.4, we have the number of smallest parts of $r-partitions$ of n having k as a smallest part is

$$\sum_{i=0}^{\infty} p_{r-1-i}\left[n-(k-1)r-1-i\right]+\beta \text{ where } \beta=\begin{cases}1 & \text{if} \quad \frac{n}{r}=k \\ 0 & \text{otherwise}\end{cases}$$

The generating function for the number of smallest parts of $r-partitions$ of n is

$$\sum_{n=1}^{\infty} r-spt(n)q^n=\sum_{k=1}^{\infty}\sum_{i=0}^{\infty}\frac{q^{r-1-i+(k-1)r+1+i}}{(q)_{r-1-i}}+\frac{q^r}{(1-q^r)}$$

$$=\sum_{k=1}^{\infty}\sum_{i=0}^{\infty}\frac{q^{kr}}{(q)_{r-1-i}}+\frac{q^r}{(1-q^r)}$$

$$=\sum_{i=0}^{\infty}\frac{(q^r+q^{2r}+q^{3r}+\ldots)}{(q)_{r-1-i}}+\frac{q^r}{(1-q^r)}$$

$$=\frac{q^r}{(1-q^r)}\sum_{i=1}^{r-1}\frac{1}{(q)_i}+\frac{q^r}{(1-q^r)}$$

$$=\frac{q^r}{\left(1-q^r\right)}\left(\sum_{i=1}^{r-1}\frac{1}{(q)_i}+1\right)$$

$$=\frac{q^r}{\left(1-q^r\right)}\left(\sum_{i=1}^{r-1}\frac{1}{(q)_i}+1\right)$$

$$=\frac{q^r}{\left(1-q^r\right)}\left(\sum_{r_1=1}^{r-1}\frac{1}{(q)_{r_1}}+1\right)$$

From theorem 1.4 and theorem1.6, we get the number of second smallest parts of $r-partitions$ of n with second least part $k \neq \lambda_1$ is

$$f_r^{\,2}(k,n)=\sum_{r=1}^{\infty}\sum_{\mu_0=1}^{\infty}\sum_{r_1=1}^{r-1}\sum_{i=1}^{\infty}p_{r_1-1-i}\left[(n-r\mu_0)-(k-1)r_1-1-i\right]$$

The generating function for the number of second smallest parts $\neq \lambda_1$ of $r-partitions$ of n

$$\sum_{r=1}^{\infty}\sum_{\mu_0=1}^{\infty}\sum_{r_1=1}^{r-1}\sum_{k=1}^{\infty}\sum_{i=1}^{\infty}\frac{q^{r_1-1-i+r\mu_0+(k-1)r_1+1+i}}{(q)_{r_1-1-i}}$$

$$=\sum_{r=1}^{\infty}\sum_{\mu_0=1}^{\infty}\sum_{r_1=1}^{r-1}\sum_{k=1}^{\infty}\sum_{i=1}^{\infty}\frac{q^{r\mu_0+kr_1}}{(q)_{r_1-1-i}}$$

$$=\sum_{\mu_0=1}^{\infty}\sum_{r=1}^{\infty}q^{r\mu_0}\left[\sum_{r_1=1}^{r-1}\sum_{k=1}^{\infty}\sum_{i=1}^{\infty}\frac{q^{kr_1}}{(q)_{r_1-1-i}}\right]$$

$$=\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\left(\sum_{r_2=1}^{r_1-1}\frac{1}{(q)_{r_2}}\right) \qquad (1.5.1)$$

From(1.3.4) the generating function for the number of second smallest parts of $r-partitions$ of n with second smallest part equal to λ_1 is

$$\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)} \quad \text{for } r=2 \qquad (1.5.2)$$

From (1.5.1) and (1.5.2) we get the generating function for the number of second smallest parts of $r-partitions$ of n which is given by

$$\sum_{n=1}^{\infty}r-spt_2(n)q^n=\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\left(\sum_{r_2=1}^{r_1-1}\frac{1}{(q)_{r_2}}\right)+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}$$

$$=\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\left(\sum_{r_2=1}^{r_1-1}\frac{1}{(q)_{r_2}}+1\right) \qquad (1.5.3)$$

By induction, the generating function for the number of i^{th} smallest parts of $r-partitions$ of n is

$$\sum_{n=1}^{\infty} r-spt_i(n)q^n = \frac{q^n}{(1-q^n)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{(1-q^{r_1})}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{(1-q^{r_2})}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{(1-q^{r_{i-1}})}\left(\sum_{r_i=1}^{r_{i-1}-1}\frac{1}{(q)_{r_i}}+1\right) \blacksquare$$

1.7 Corollary:

The generating function for the number of $k's$ as smallest part of

$r-partitions$ of n is

$$q^{kr}\left[\sum_{i=1}^{r-1}\frac{1}{(q)_i}+1\right]$$

1.8 Corollary: The generating function for the number of $k's$ as smallest parts of *partitions* of n is

$$\sum_{r=1}^{\infty} q^{kr}\left[\sum_{i=1}^{r-1}\frac{1}{(q)_i}+1\right]$$

1.9 Corollary: The generating function for the number of $r-partitions$ of n having i distinct integers is

$$\sum_{r_1=1}^{\infty}\frac{q^{r_1}}{(1-q^{r_1})}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{(1-q^{r_2})}\cdots\sum_{r_i=1}^{r_{i-1}-1}\frac{q^{r_i}}{(1-q^{r_i})}$$

1.10 Corollary: The generating function for the number of smallest parts of $r-partitions$ of n which are multiples of c is

$$\sum_{n=1}^{\infty} c(r-spt_i(n))q^n = \frac{q^{cr}}{(1-q^{cr})}\left[\sum_{r_1=1}^{r-1}\frac{1}{(q)_{r_1}}+1\right]$$

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On Semicircular Extreme-value distribution

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Abstract

This paper is aimed at construction of Semicircular Extreme-value distribution for modeling semicircular data by applying simple projection method on Extreme-value distribution. It is extended it to the l-axial Extreme-value distribution by a simple projection for modeling any arc of arbitrary length.

Keywords: characteristic function, semi circular models, l-axial data, projection, trigonometric moments.

AMS Subject Classification: 60E05, 62H11

1. INTRODUCTION

In some of the cases the directional / angular data does not require full circular models for modeling, this fact is noted in Guardiola (2004), Jones (1968) and Byoung et al(2008). For example, when a sea turtle emerges from the ocean in search of a nesting site on dry land, a random variable having values on a semicircle is well sufficient for modeling such data. Similarly, when an aircraft is lost but its departure and its initial headings are known, a semicircular random variable is sufficient for such angular data. And few more examples of semicircular data is available in Ugai et al (1977).

Guardiola (2004) obtained the semicircular normal distribution by using a simple projection and Byoung et at (2008) developed a family of the semicircular Laplace distributions for modeling semicircular data by simple projection, Phani et al (2013) constructed some semicircular distributions by applying inverse stereographic projection. In this paper the **Semicircular Extreme-value distribution** (**SCEVD**) is developed by projecting Extreme-value distribution over a semicircular segment. The graphs of the density and distribution functions for various values of parameters are plotted. The characteristic function is also derived. This model is extended to l-axial model.

2. SEMICIRCULAR EXTREME-VALUE DISTRIBUTION

A random variable X on the real line is said to have Extreme-value Distribution with location parameter γ and scale parameter $\lambda>0$, if the probability density function and cumulative distribution function of X for $x,\gamma\in\mathbb{R}$ and $\lambda>0$ are given respectively by

$$f(x)=\frac{1}{\lambda}\exp\left(\frac{-(x-\gamma)}{\lambda}\right)\exp\left(-\exp\left(\frac{-(x-\gamma)}{\lambda}\right)\right),$$

where $\lambda>0$, and $\gamma,\ x\in\mathbb{R}$ (2.1)

$$F(x)=\exp\left(-\exp\left(\frac{-(x-\gamma)}{\lambda}\right)\right)\quad . \tag{2.2}$$

Simple projection is defined by

mapping $x=v\tan(\theta)$ or $\theta=\tan^{-1}\left(\frac{x}{v}\right),\ v>0\in\mathbb{R}$.

Application of this simple projection on Extreme – value distribution results to a Semicircular Extreme-value distribution.

Definition:

A random variable X_{SC} on the Semicircle is said to have the Semicircular Extreme-value distribution with location parameter μ scale parameter $\sigma>0$ denoted by **SCEVD**(σ,μ), if the probability density and the cumulative distribution functions are respectively given by

$$g(\theta)=\frac{1}{\sigma}\sec^2(\theta)\exp\left(-\frac{(\tan(\theta)-\mu)}{\sigma}\right)\exp\left\{-\exp\left(-\frac{(\tan(\theta)-\mu)}{\sigma}\right)\right\},$$

$$\text{where, }\mu=\frac{\gamma}{v},\ \sigma=\frac{\lambda}{v}>0 \tag{2.3}$$

$$G(\theta) = \exp\left\{-\exp\left(-\frac{\left(\tan(\theta)-\mu\right)}{\sigma}\right)\right\}, \text{where } \sigma > 0,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2} \tag{2.4}$$

Hence the proposed new model **SCED** (σ, μ) is a Semicircular model.

Graphs of the probability density and cumulative distribution functions of the Semicircular Extreme-value distribution for various values of σ and μ

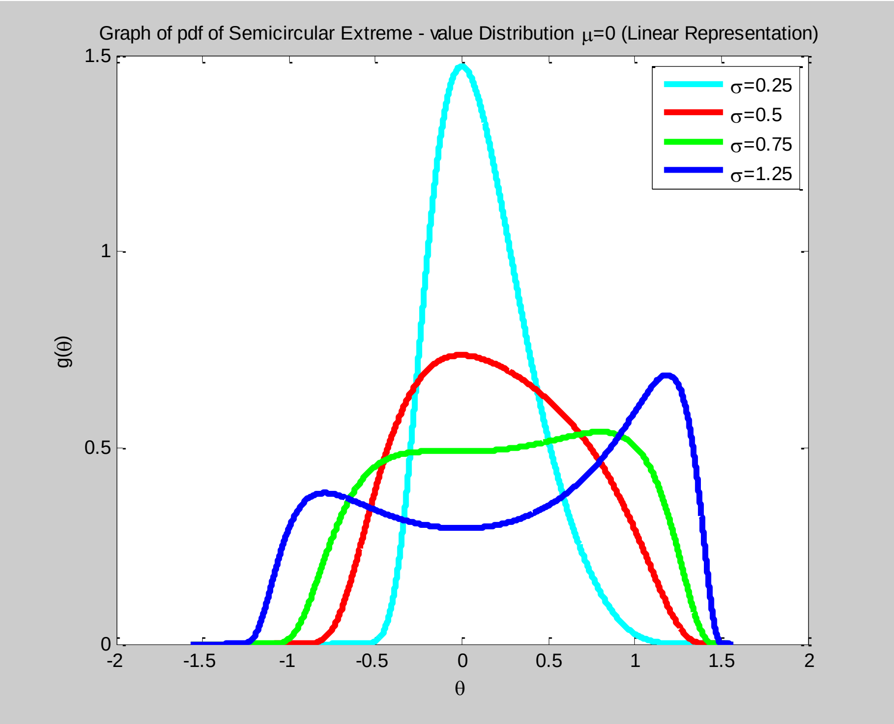

Fig1. Graph of pdf of Semicircular Extreme – value Distribution (Linear Representation)

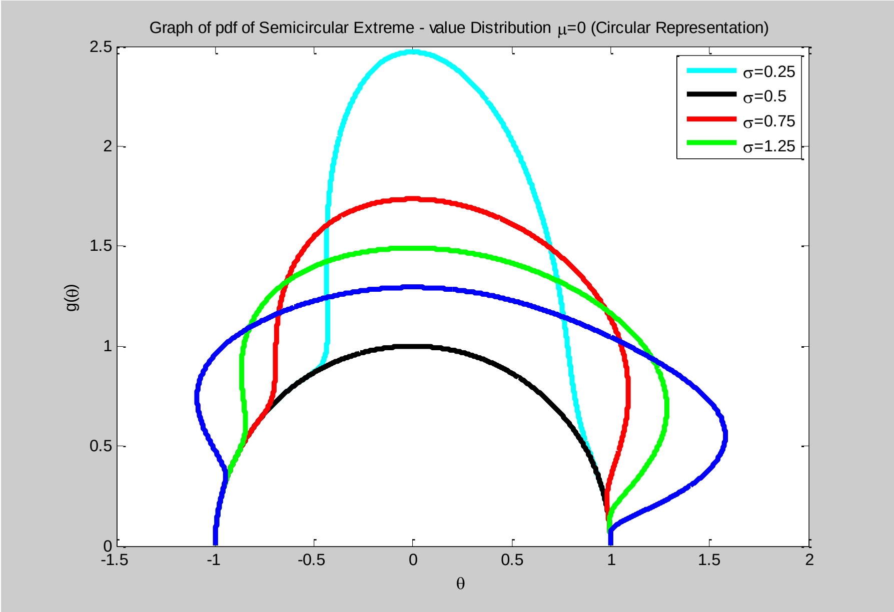

Fig2. Graph of pdf of Semicircular Extreme – value Distribution (Circular Representation)

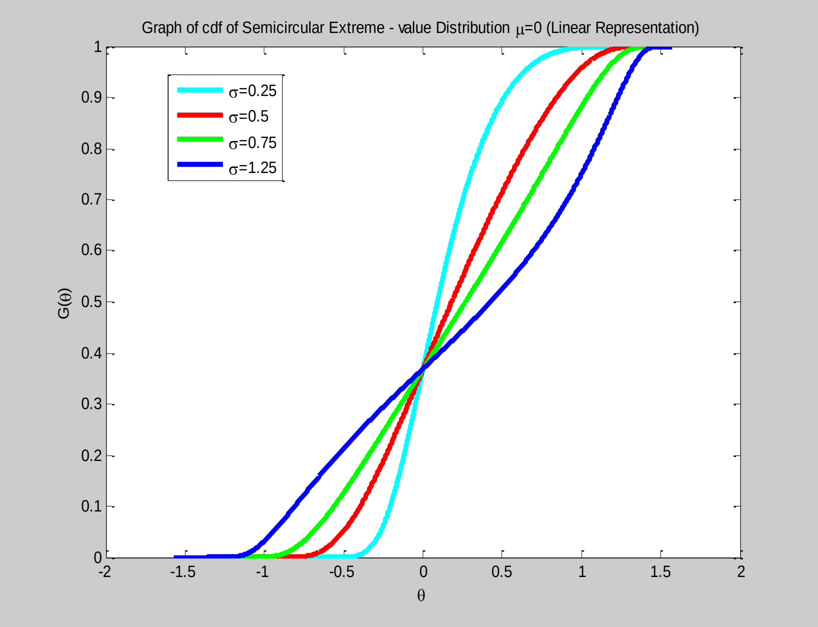

Fig3. Graph of cdf of Semicircular Extreme – value Distribution

3. The Characteristic function of Semicircular Extreme-value distribution

The characteristic function of a semicircular model with probability density function $g(\theta)$ is defined as

$$\Phi_{X_{SC}}(p) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{ip\theta} g(\theta)\, d\theta \quad , p \in \mathbb{Z}.$$

$$\Phi_{X_{SC}}(p) = \frac{1}{\sigma} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{ip\theta} \sec^2(\theta) \exp\left(-\frac{(\tan(\theta)-\mu)}{\sigma}\right) \exp\left\{-\exp\left(-\frac{(\tan(\theta)-\mu)}{\sigma}\right)\right\} d\theta \quad , p \in \mathbb{Z} \qquad (3.1)$$

As the integral cannot be obtained analytically, MATLAB techniques are applied for the evaluation of the values of the characteristic function. Population characteristics are also studied using first two trigonometric moments.

Graphs of the characteristic function of Semicircular Extreme -value distribution for various values of parameter are plotted.

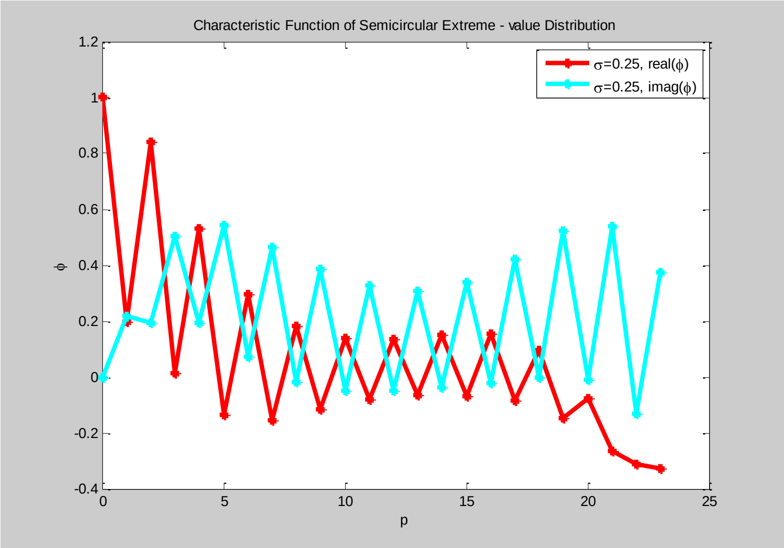

Fig4. Graph of Characteristic function of Semicircular Extreme – value Distribution

The expressions for mean direction, resultant length, circular variance, circular standard deviation, central trigonometric moments, skewness and kurtosis for circular distributions are available in Mardia and Jupp (2000). These characteristics for the Semicircular Extreme - value distribution are also based on their respective trigonometric moments and can be expressed in terms of trigonometric moments α_p and β_p which are presented here.

From the population characteristics, it can be observed that with increasing value of scale parameter σ , the circular variance gradually decrease, the distribution is negatively skewed and remained platykurtic

Table 3.1 Population Characteristics of Semicircular Extreme – value model

	$\sigma = 0.25$	$\sigma = 0.5$	$\sigma = 0.75$	$\sigma = 1.25$
Trigonometric Moments				
α_1	0.1995	0.1074	-0.0183	-0.1525
α_2	0.8426	0.6299	0.4812	0.1586
β_1	0.2176	0.3461	0.3727	0.2671
β_2	0.1926	0.2203	0.2488	0.3660
Resultant Length				
ρ_1	0.2952	0.3624	0.3732	0.3076
ρ_2	0.8643	0.6673	0.5417	0.3989
Mean Direction μ_0	0.8288	1.2700	-1.5218	-1.0518
Circular Variance v_0	0.7048	0.6376	0.6268	0.6124
Circular Standard Deviation				
σ_0	1.5622	1.4249	1.4041	1.5356
	0.5400	0.8994	1.1074	1.3558
Central Trigonometric Moments				
α_1^*	0.2952	0.3624	0.3732	0.3076
α_2^*	0.1188	-0.3947	-0.5032	-0.3958
β_1^*	0	0	0	0
β_2^*	-0.8561	-0.5381	-0.2005	-0.0493
Skewness γ_1^0	-1.4467	-1.0568	-0.4040	-0.0856
Kurtosis γ_2^0	0.2239	-1.0131	-1.3299	-0.8442

4 Extension to l -axial distribution

The newly proposed model is extended to the l -axial distribution, which is applicable to any arc of arbitrary length say $2\pi/l$ for $l = 1, 2, \ldots,$. So it is desirable to extend the Semicircular Extreme-value distribution to its l – axial model. To construct the l -axial Extreme-value distribution, the density function of Semicircular Extreme-value distribution is considered and the transformation $\phi = 2\theta/l$, $l = 1, 2, \ldots,$ is used. The probability density function of ϕ is given by

$$g(\phi) = \frac{l}{2\sigma} \sec^2\left(\frac{l\phi}{2}\right) e^{-\frac{\tan\left(\frac{l\phi}{2}\right)}{\sigma}} e^{-e^{-\frac{\tan\left(\frac{l\phi}{2}\right)}{\sigma}}}$$

$$\text{where} \quad -\frac{\pi}{l} < \phi < \frac{\pi}{l} \qquad (4.1)$$

Case (1) When $l = 2$, the probability density function (4.1) is the same as the probability density function of Semicircular **Extreme-value distribution .**

Case (2) When $l = 1$, the probability density function (4.1) is the same as that of **Stereographic Extreme-value Distribution** [Phani et al., 2012] which is circular distribution.

If X has Extreme-value with scale parameter λ and location parameter $\gamma = 0$, then $Y = e^X$ has a **log-Extreme-value distribution** with the same parameters. So, using $y = v\tan(\theta)$, a probability density function is

$$g(\theta) = \frac{2}{\lambda\sin(2\theta)} e^{-\frac{1}{\lambda}(\log(v\tan(\theta)))} e^{-e^{-\frac{1}{\lambda}\left(\log(v\tan(\theta))\right)}} \quad 0 < \theta < \frac{\pi}{2}, \qquad (4.2)$$

Suppose $\phi = \frac{4\theta}{l}$, $l = 1, 2, \ldots,$ then ϕ has a probability density function ,

$$g(\phi) = \frac{1}{2\lambda\sin\left(\frac{l\phi}{2}\right)} e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{l\phi}{4}\right)\right)\right)} e^{-e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{l\phi}{4}\right)\right)\right)}}, \ 0 < \theta < \frac{2\pi}{l} \qquad (4.3)$$

It is said that ϕ follows the l **- axial log-Extreme-value distribution**.

Case (1): When $l = 1$,

$$g(\phi) = \frac{1}{2\lambda\sin\left(\frac{\phi}{2}\right)} e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{\phi}{4}\right)\right)\right)} e^{-e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{\phi}{4}\right)\right)\right)}}, \ 0 < \theta < 2\pi \qquad (4.4)$$

It is called as a **Circular log-Extreme-value distribution**.

Case (2): When $l = 2$,

$$g(\phi) = \frac{1}{2\lambda\sin(\phi)} e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{\phi}{2}\right)\right)\right)} e^{-e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{\phi}{2}\right)\right)\right)}}, \ 0 < \theta < \pi \qquad (4.5)$$

It is called as a **Semicircular log-Extreme-value distribution**.

Case (3): When $l = 4$,

$$g(\phi) = \frac{1}{2\lambda\sin(2\phi)} e^{-\frac{1}{\lambda}(\log(v\tan(\theta)))} e^{-e^{-\frac{1}{\lambda}\left(\log(v\tan(\theta))\right)}}, \ 0 < \theta < \frac{\pi}{2} \qquad (4.6)$$

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Stability and Validity Indicating Assay Method for Estimation of Multi drug components of Etophylline and Theophylline Anhydrous – Forced Degradation Studies

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Abstract

A very simple, more selective, high linear, most precise along with accurate HPLC process is advanced also validated to rapid assay of Etophylline and theophylline which are in Bulk and in pharma oral system formulation. Chromatographic investigation is subjected by utilization of Cosmosil C18 5 µm (4.6 x 150 mm) or equivalent column at ambient temperature. UV detection wavelength is noted at 254nm, sample passed into system as 20µl. Rate of flow is 1.0ml/min. Iin this process utilized mobile phase as 0.05M KH_2PO_4 whose pH is adjusted to 3 by phosphoric acid as 95V/V along with acetonitrile as 5V/V. Rt to Etophylline is ± 12 minutes, run time is 19 min. Average percentage recovery is in the range of 99-102%. The method is favorably applied to periodic quality control analysis of pharma formulation.

Key Words**:** Etophylline, Theophylline, UV Spectroscopy, Suitability, Recovery, Precise.

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1. Introduction

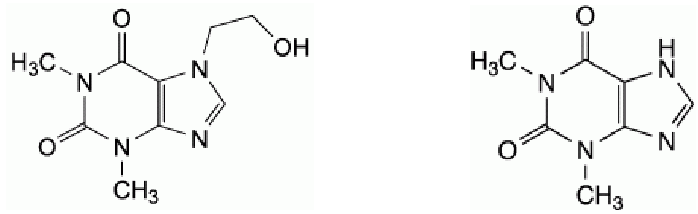

Fig:1 Structure of Etophylline and Theophylline Anhydrous

Etophylline also represented as β- Hydroxyethyl Theophylline which is stored in very tight, light antagonistic container. Proposed drug is white, more crystalline powder. This drug is more soluble in water, in rare cases low soluble in alcohol. Theophylline anhydrous is white also highly crystalline powder. This drug is a little soluble in water, faintly soluble in ethanol. It subjects to dissolution in various solutions like alkali hydroxides, ammonia along with various mineral acids. Tablets includes Etophylline, theophylline are more effective ingredients. For this chemical name is 7-(2-hydroxyethyl)-1, 3-dimethyl-3, 7dihydro-1H-purine-2, 6-dione[1]. THEO indicated to chronic obstructive problems of air ways, COPD bronchial asthma, child apnea[2]. Various processes[3-6]are characterized to determine ETO individually also in combination by different drugs. Procedure include serum RP-HPLC[7],TLC methods[8-9]along with derivative spectrophotometric process[10]. Devang.N.A et.al.,[11] proposed both Etophylline and Theophylline in injectable forms. Quantification is borne by utilizing Silica gel column, which is packed by octadecylsilane, 2.1 mm × 100 mm, i.d, 1.7µm particle size. In this process proposed segregation is carried in isocratic mode which consists a mixture of 0.05 M Sodium Acetate along with acetonitrile as 90:10 composition, pH is noted as 4.5 flow rate is fixed at 0.3 ml / min. Detected wavelength as 270nm. Retention time for Theophylline is 2.368 min and for Etophylline is 3.129 min. Recovery to proposed drugs are in the range of 99.84 - 100.89%. S.Nagajyothi et.al.,[12] proposed Etophylline as ETO ,Theophylline as THEO in bulk along with solid dosage forms. Wavelengths to both selected drugs are 271nm for ETO, 273nm THEO. V. D. N. Kumar Abbaraju et.al.,[13] proposed Pholcodine at rate of flow as 1.0ml/min column utilized Symetry or Luna C8 5µm (4.6 x 150 mm). Mobile phase consists of methanol as 70v/v, water as 20v/v along with acetonitrile as 10v/v. For this process wavelength noted as 220nm, sample passed in to column as 100µl. Run time as 10minutes, flow rate is 1.5ml. % R.S.D for this drug is 0.4. LOD is identified as 15µg/ml and LOQ is noted at 25µg/ml. Mean percentage recovery is 80%. Finally by studying above literature developed process is simpler, superior to other methods reported in the literature.

2. EXPERIMENTAL

2.1 Instrumentation

For this investigation peak HPLC containing LC 20AT pump along with changeable wavelength programmable UV-Visible detector, Rheodyne injector is engaged for through check. chromatographic examination and determination is accomplished over Cosmosil C18 5 µm (4.6 x 150 mm) or equivalent column. Degassing of mobile phase is done utilizing Loba ultrasonic bath sonicator. Denwar analytical balance is operated inorder to weigh materials.

2.2 Chemicals and solvents

To this analysis grinded sample content of Etophylline (β-Hydroxyethyl Theophylline), product Theophylline anhydrous is poised. KH_2PO_4, Phosphoric acid, Acetonitrile, water along with buffer utilized are HPLC grade those are procured from Merck, India.

2.3 Mobile phase

A mixture of 0.05M KH_2PO_4 which pH is adjusted to 3 by phosphoric acid taken as 95v/v and acetonitrile 5v/v is utilized.

2.4 Ground work for solutions

Mobile phase preparation

Exactly 6.8045g of KH_2PO_4 is taken in 500ml beaker which contains water, after that adjusted pH to 3 by phosphoric acid. Transferred same solution in 1000ml standard flask. After that subjected dilution to suitable volume by water. Latter mixed with Methanol in the ratio which is provided to dilute fragments. Filtered and run over line in ratio likely by methanol.

Standard Solution of Etophylline and Theophylline

Exactly 67.00 mg β-Hydroxyethyl Theophylline RS standard is taken in 100ml standard flask this is noted as solution A. After that subjected to dissolution latter diluted to suitable capacity by methanol. In order to prepare theophylline, exactly 67.00mg Theophylline Anhydrous taken in 250ml flask. To this about 10ml of solution A is pipetted and transferred in 250ml. After that subjected to dissolution and diluted to suitable volume by mobile phase. Finally, filtered through 0.22 µm filter by separating first 5ml filtrate.

Preparation of Specimen

Exactly 10.00gm fragment is taken in 100ml standard flask. To this added 50.00ml mobile phase and subjected to sonication for mixing. After that diluted to volume by above said mobile phase. Subjected to filtration by 0.45µm filter. Finally discard first 5ml of filtrate.

3. Development of Method

Various optimized circumstances are followed to conclude in bulk samples along with its combined assorted oral tablet formulations. Obtained chromatograms of both standard and sample are recorded. Spectrum to 10ppm solution of Theophen compound elixir is documented separately over spectrophotometer. Finally peak of maximum absorbance wavelength is 254nm is recognized. Expected separation along with shapes of peak is recognized over Cosmosil C18 5 µm (4.6 x 150 mm) or equivalent column. To identify rate of flow various mobile phase are alternated as 0.5-1.5 ml/min to get excellent disengagement. Finally ideal rate of flow is founded as 1.0ml/min inorder to elute analyte. Volume injected as 20 µl. Approx. Rt. to Theophylline Anhydrous is$\pm$ 12 minutes.

4. Validation Of Proposed Method

Different parameters designed to validation are specificity, linearity, precision, accuracy, robustness, system suitability, limit of detection, limit of quantification and solution stability.

4.1 Specificity

Both solvent along with inactive drug solutions won't accommodate any constituents, those co-elute by Etophylline also anhydrous of Theophylline. Purity in peak values by photo diode-array investigation showed that both peaks are pure indicates that angle of purity is inferior to angle of threshold. Solutions like Etophylline and Theophylline anhydrous passed by utilizing required circumstances which is proposed in analysis method are stable under UV light vulnerability. There are no other components observed for co-elute by Etophylline along with Theophylline anhydrous peaks. Finally results of purity peak demonstrate both Etophylline and Theophylline anhydrous deliberated as spectrally pure. Finally from observations the chromatograms obtained are shown in the figure 2 to 5, peak purity results are shown in figure 6 to 9.

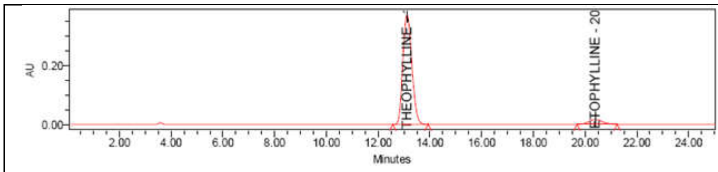

Fig: 2 Chromatogram 1: Drug active – Peak due to Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous

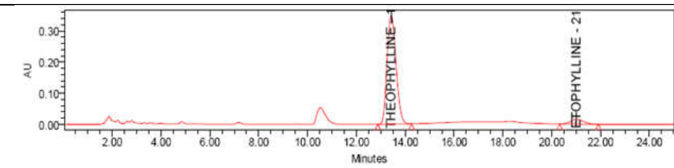

Fig: 3 Chromatogram 2: Product – Peak due to Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous

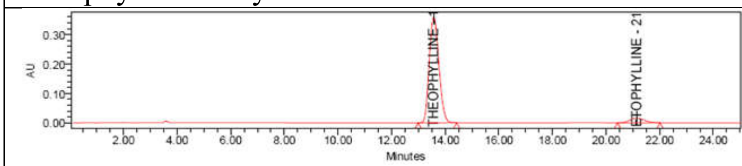

Fig: 4 Chromatogram 3: Active – UV stress: Peak due to Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous

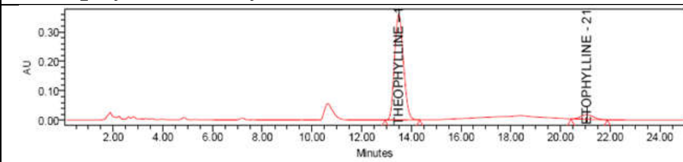

Fig: 5 Chromatogram 4: Product – UV stress: Peak due Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous

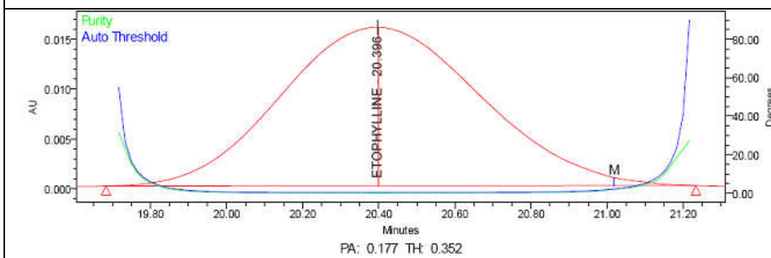

Fig: 6 Peak Purity 1: Drug Active Etophylline (β Hydroxyethyl Theophylline) Purity Angle <Threshold: 0.177 < 0.352

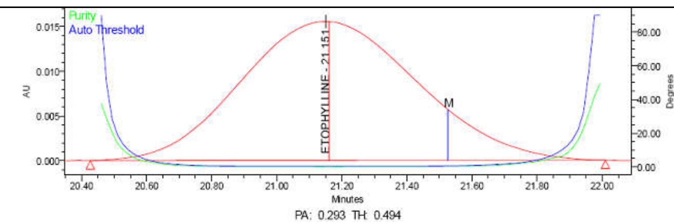

Fig: 7: Peak Purity 2: Drug Active Etophylline (β Hydroxyethyl Theophylline) Purity Angle <Threshold: 0.293 < 0.494

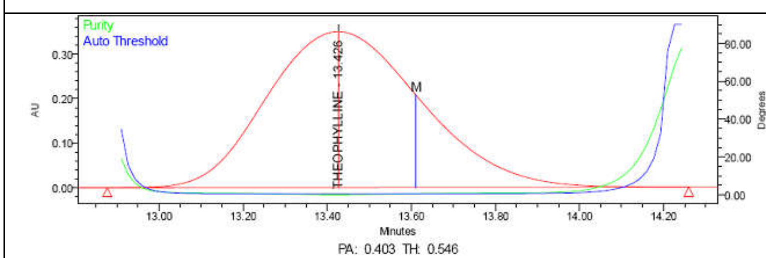

Fig: 8: Peak Purity 3: _Theophylline Peak Purity Angle <Threshold: 0.403 < 0.546

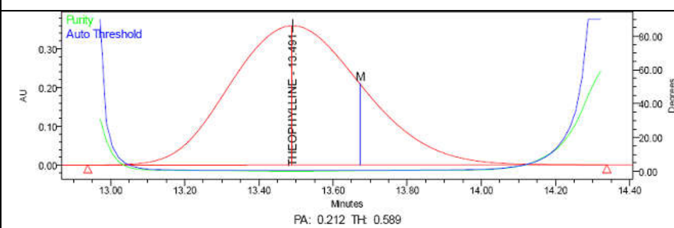

Fig: 9: Peak Purity 4: Theophylline UV Stressed Drug Product

Forced Degradation Studies:

These studies are performed by considering various sample preparations which includes specificity by forced degradation, preparation of sample, construction of placebo, 1.0N Hcl stressed sample, sample of 1.0N NaOH stressed, sample of 3.0% w/v Hydrogen Peroxide stressed, stressed sample of neutral, sample which is exposed to UV light, sunlight along with sample which is also stressed by thermal denoted as dry heat. Finally these studies are concluded as total degradation products were taken out from Etophylline (β-Hydroxyethyl Theophylline), product Theophylline anhydrous peak also each other. Peak purity angle is minimum to purity threshold by waters empower-2 software. Finally it is concluded samples are undergoing degrading in both circumstances. Also found that in sunlight condition Etophylline (β-Hydroxyethyl Theophylline), product Theophylline anhydrous peaks are subjected to degradation normally, for remaining no degradation is identified. Anyhow total degradation products are separated very well by Etophylline (β-Hydroxyethyl Theophylline), product Theophylline anhydrous peak along with each other. Etophylline (β-Hydroxyethyl Theophylline), product Theophylline anhydrous peak and impurities peaks are more pure. Due to this reason related substances method is considered more specific & highly stability indicating.

4.2 System Suitability

% RSD of response in peak is mainly as a result of Etophylline along with Theophylline anhydrous to total six simulate injections to each and every prerequisite be minimum as 2.0 %. Total six replicate injections of working standard sample solution are passed by considering analysis of method. % RSD to peak responses are measured. Investigative system accumulate by prerequisite which is particularized by this parameter. Finally results obtained are tabulated in the table: 1.

Sample	Etophylline (β-Hydroxyethyl Theophylline)	Theophylline
1	583050	8322581
2	584320	8333629
3	584644	8339267
4	585002	8334340
5	585018	8329352
6	584723	8350223
Mean	584460	8334899
% RSD	0.1	0.1

Table:1 System Suitability results

4.3 Linearity

Correlation coefficient for regression line to β-Hydroxyethyl Theophylline along with Theophylline anhydrous are very nearer to value 0.99. Y-intercept for line to assess z lowering along specified range only when + 5 > z > - 5. Total five solutions which contains 50, 75, 100, 125, and 150 % of β-Hydroxyethyl Theophylline along with Theophylline anhydrous relative for working strength, those are designed after that injected as per procedure of analysis. By values obtained a linear regression curve is assembled, after that both R^2 along with computation values are determined. R^2 to both β-Hydroxyethyl

Theophylline and Thephylline anhydrous are equal to 1.0. Both plots constructed are appeared as straight lines finally value of z for both were 0.49 and 1.99. Therefore, from total results obtained concluded that analysis for Etophylline and Theophylline in Theophen Compound Elixir are noted as linear. Results obtained are tabulated in table.2 and linearity curve is shown in the fig 10.

Sample Number	Concentration	Response 1	Response 2	Average Response
50%	0.134000	4261261.0000	4264119.0000	4262690.0000
75%	0.201200	6293041.0000	6296606.0000	6294823.5000
100%	0.268000	8422748.0000	8422146.0000	8422447.0000
125%	0.335200	10447659.0000	10467901.0000	10457780.0000
150%	0.402000	12438737.0000	12435040.0000	12436888.5000

Table:2 Linearity results

CALIBRATION CURVE: y = Bx + A, R^2 = coeff. of Correlation

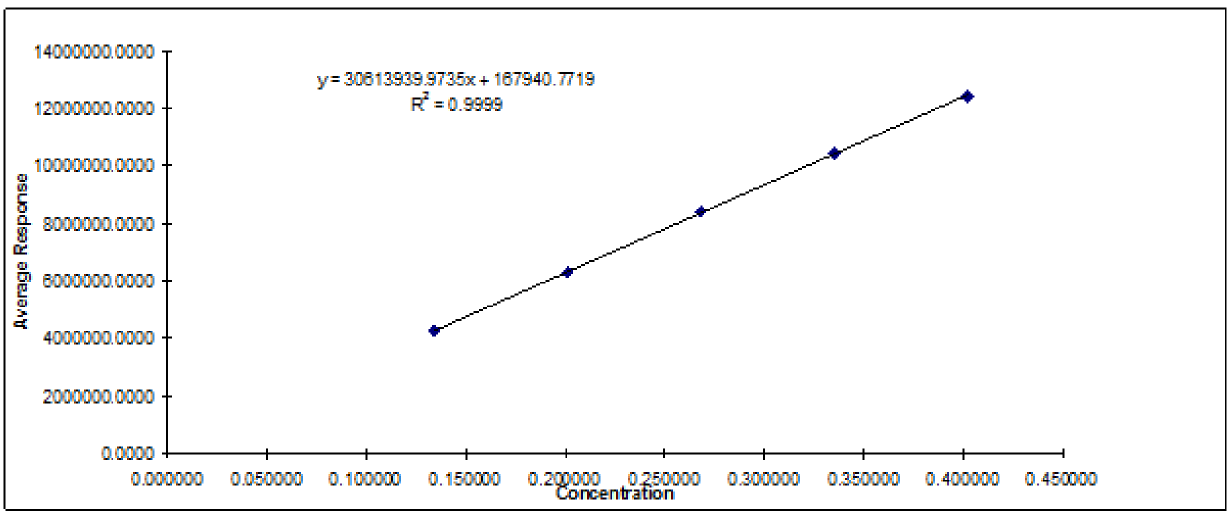

Fig: 10 Linearity curve

4.4 Accuracy

To assess this parameter percentage recovery of active substance to each solution constructed is in the range of 95.0 – 105.0 % to actual amount. collected samples are subjected to weighing in various 100ml standard flasks those consists of strengths β-Hydroxyethyl Theophylline after that Theophylline anhydrous are 50, 75, 100, 125 and 150 % of β-Hydroxyethyl Theophylline also Theophylline anhydrous those are analogues to working stregnths. Represented samples are passed in duplicate as per analysis process. By values of accuracy obtained, percentage recovery values to both β-Hydroxyethyl Theophylline along with Theophylline anhydrous satisfies acceptance criteria to this parameter over range of 50 % - 150 %. Finally, obtained values are noted in table 3 and 4.

Sample %	Sample weighed (g)	Theoretical % Active	Actual % Active	% Recovery	Average % Recovery
50	5.0005	4.95	4.9	99.0	99.0
			4.9	99.0	
75	7.5002	7.43	7.3	98.3	98.3
			7.3	98.3	
100	10.0005	9.85	9.8	99.5	100.0
			9.9	100.5	
125	12.5004	12.38	12.3	99.4	99.8
			12.4	100.2	
150	15.0003	14.85	14.8	99.7	99.7
			14.8	99.7	

Table 3: Etophylline (β-Hydroxyethyl Theophylline) - Percentage recovery results

Sample %	Sample weighed (g)	Theoretical % Active	Actual % Active	% Recovery	Average % Recovery
50	5.0005	50.8	51.6	101.6	101.7
			51.7	101.8	
75	7.5002	76.2	76.2	100.0	99.9
			76.0	99.7	
100	10.0005	101.6	101.4	99.8	100.0
			101.8	100.2	
125	12.5004	127.0	126.3	99.4	99.5
			126.5	99.6	
150	15.0003	152.4	150.2	98.6	98.6
			150.2	98.6	

Table 4: Theophylline Anhydrous - Percentage recovery results

4.5 Method precision

4.5.1 Repeatability

For this parameter % RSD is due to β-Hydroxyethyl Theophylline and Theophylline anhydrous strength to total six samples is found that 2.0 %. % RSD due to β-Hydroxyethyl Theophylline along with Theophylline anhydrous strength assay meets necessitates to repeatability founded as 0.9 % and 1.0 % for both drugs. Results obtained are noted in table 5.

Sample number	Results (mg/30ml)	
	Etophylline (β-Hydroxyethyl Theophylline)	Theophylline Anhydrous
1	9.8	100.2
2	9.8	99.9
3	9.8	99.7
4	9.6	97.6
5	9.8	99.8
6	9.7	99.1
Mean	9.7	99.4
% RSD	0.9	1.0

Table 5: Repeatability results

4.5.2 Intermediate Precision

By utilizing proposed parameter % RSD is mainly due to β-Hydroxyethyl Theophylline along with Theophylline anhydrous strength to total six samples is corresponding to 2.0 %. Finally ungenerous values achieved over both repeatability along with intermediate precision should not vary to value 3.0 %. Total six various sample preparations are assayed as per protocol of analysis of method. % RSD is mainly due to β-Hydroxyethyl Theophylline along with Theophylline anhydrous strength assay meets possible concern to Intemediate Precision at 0.6 % and 1.0 %. % difference of mean to both drugs in the intermediate precision and repeatability both comply as they varied as 1.0 % and 1.4 % respectively. Finally values are tabulated in table

Sample number	Results (mg/30ml)	
	Etophylline (β-Hydroxyethyl Theophylline)	Theophylline Anhydrous
1	9.7	97.5
2	9.7	98.5
3	9.8	98.3
4	9.9	98.5
5	9.8	97.3
6	9.9	98.1
Mean	9.8	98.0
% RSD	0.6	1.0

Table 6: Intermediate precision results

5. Stability Of Solution

This aspect provided values showed that there no loss of active in testing sample preparation while standing at a period of 24 hours. % RSD is mainly due to β-Hydroxyethyl Theophylline along with Theophylline anhydrous content to samples is noted as 1.5 %. Stability to solution mean of β-Hydroxyethyl Theophylline and theophylline anhydrous varies by attain intermediate repeatability mean by 0.50 %.

4.6 RANGE

This parameter is derived as per values obtained by accuracy, range to assay of β-Hydroxyethyl Theophylline along with Theophylline anhydrous is 5.0 – 15.0 mg per 30ml and 50.0 – 150.0 mg per 30 ml respectively, this denotes 50 % to 150 % to working strength.

4.7 Limit of Detection and Limit of Quantification

These parameters to proposed drugs are measured by utilizing standard equation proposed by ICH guidelines. For this parameter results obtained to proposed drug β-Hydroxyethyl Theophylline is 2.3, 7.2 and for Theophylline anhydrous is 1.98, 6.0

5. Results And Discussion

To optimize the parameters, several mobile phase compositions were tried and concluded with Cosmosil C18 5 µm (4.6 x 150 mm) or equivalent column. A satisfactory separation and good peak symmetry was found in a mixture of 0.05M KH_2PO_4 whose pH is adjusted to 3 by phosphoric acid as 95V/V along with acetonitrile as 5V/V. Flow rate is identified as 1.0ml/min. proved to be better than the other mixtures in terms of resolution and peak shape. Wavelength for detection is 254nm. Rt values for both proposed drugs is ± 12 minutes. To various parameters we are performed system suitability test after that applied to representative chromatograms for various parameters. Results obtained were acceptable limits. Therefore, system meets suitable criteria. Calibration curve was obtained to series of strengths those are in the range of 0.6-1.4µg/ml, it was founded as linear. Total five points graph is constructed after that identified R^2 to both β-Hydroxyethyl Theophylline and Theophylline anhydrous as 0.0007. Plot is straight line and z is 0.6 for both drugs. Forced degradation studies are also studied by preparing different solutions noted in the article. All these factors lead to the conclusion that the proposed method is more accurate, most precise, very simple, sensitive along with rapid also applied more successfully to estimate both β-Hydroxyethyl Theophylline and Theophylline anhydrous in bulk and in pharmaceutical formulations as oral dosage without interference and with good sensitivity.

6. Declaration On Validity Of The Method

The method for the assay of β-Hydroxyethyl Theophylline and Theophylline Anhydrous comply with the requirements for linearity, method precision and accuracy for β-Hydroxyethyl Theophylline and Theophylline Anhydrous across the range of 50 % to 150 %. The Cleaning Validation method is proven to be valid and the validation test results show that the method complies with the validation requirements. Therefore, the method is acceptable as valid.

7. Acknowledgement

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Stability and Validity Indicating UV Spectrophotometric Rapid Assay Method for The Estimation of Promethazine Theoclate

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Abstract

Asimple, selective, linear, precise and accurate UPLC method was developed and validated for rapid assay of Promethazine Theoclatein Bulk and Pharmaceutical tablet Formulation. The chromatographic analysis was performed on a Jasco Model: V-630, with a fixed bandwidth 2 nm and a pair of 1cm matched quartz cell at ambient temperature. The UV detection wavelength was observed at 252nm and injected sample is 20µl. Isocratic elution at a flow rate of 1.0ml/min was employed. The Mobile phase consisting of 0.05M KH_2PO_4 (to pH 3 with Phosphoric acid) 95V/V and Acetonitrile 5V/VMethanol and waterin the ratios of 50v/v and 50v/v. The Rt for Etophyllinewas $\pm$ 12 minutes approximately is identified. The Average percentage recovery of the method was in the range of 99-102%. The method was validated as per the ICH guidelines. The method was successfully applied for routine quality control analysis of pharmaceutical formulation.

Key Words: Promethazine Theoclate, UV Spectroscopy, Suitability, Recovery, Precise

1. Introduction

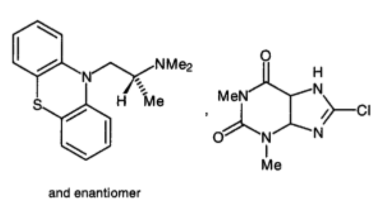

Fig.- 1: Structure of Promethazine Theoclate

A white or almost white powder, very soluble in water, sparingly soluble in ethanol (96%), practically insoluble in ether. The molecular formulae is $C_{17}H_{20}N_2S,C_7H_7ClN_4O_2$. It is a generic medication under many brand names globally.[1]Promethazine, a phenothiazine derivative, is structurally different from neuroleptic phenothiazines, with similar effects.[2]Promethazine is a first-generation antihistamine.[3] It is taken by mouthor by injection into muscle.[3]It is in the phenothiazine family of medications.[3]It is used to treat allergies, trouble sleeping, and nausea. It may also be used for sedating people who are agitated or anxious.[4,5] It is unclear if use during pregnancy or breastfeeding is safe for the baby.[3,4] Use by injection into a vein is not recommended due to potential skin damage.[4] Promethazine made in 1940s by a team of scientists from Rhône-Poulenc laboratories.[6] The cost by mouth formulation is less than 0.20 USD per dose as of 2018.[7] Another notable use of promethazine is a local anesthetic, by blockade of sodium channels.[8]It is a chiral compound, occurring as mixture of enantiomers.[9]It was first synthesizedin the early 1940s.[10] The team was seeking to improve on diphenhydramine; the same line on medical chemistry led to the creation of chlorpromazine.[11]Thumma S et.al.,[12] proposed PMZ in hot-melt extruded. Separation was achieved on a 150 mm x 4.6 mm i.d., 3 microm particle size, C8(2) column by acetonitrile-25mM phosphate buffer (pH 7.0), 50:50 (v/v) as mobile phase at a flow rate of 1 mL min^{-1}. UV detection is 249 nm. Jagdish Kakadiya et.al.,[13] proposed PMZ and PCM. The separation was carried out on Hyperchrom ODS-BP (4.6 mm X 250 mm, 5 µm) column using Methanol: Water with 1% TEA in the ratio of 30:70 v/v as eluent. The flow rate was 1ml/min and effluent was detected at 250 nm. Rt were 3.10 and 1.72min.Linear range is 2-10 µg/ml for PMZ and 50-250 µg/ml PCM. % recoveries were 99.00-100.33% and 99.56-101.28%. V. D. N. Kumar Abbaraju et.al.,[14] proposed Flucloxacillin Sodium in tablet Formulation. Flow rate is 1.0ml/min was employed on symmetry Bondpeak C18 10 µm (3.9 x 300 mm). The mobile phase consisted of Acetonitrile and 0.0.1M Phosphate buffer in the ratio of 35:65/v/v. Wavelengthis 273nm and 10µl sample is injected. The Rt is 1.5min. % RSD for precision and accuracy of method is 0.1%.

2. Experimental

2.1 Instrumentation,

Peak HPLC containing LC 20AT pump and variable wavelength programmable UV-Visible detector and Rheodyne injector was employed for investigation. The chromatographic analysis was performed on a Jasco Model: V-630, with a fixed bandwidth 2 nm and a pair of 1cm matched quartz cell. Degassing of the mobile phase was done using a Loba ultrasonic bath sonicator. A Denwar analytical balance was used for weighing the materials.

2.2 Chemicals and solvents

The reference powdered sample Content of Theophen Compound tablets Anhydrous is collected. The Formulation was procured from the local market. KH2PO4, Phosphoricacid, Acetonitrile, water and buffer used were of HPLC grade and purchased from Merck Specialities Private Limited, Mumbai, India.

2.3 The mobile phase

The Mobile phase consisting of 0.05M KH_2PO_4 (pH 3 with Phosphoric acid) 95V/V and Acetonitrile 5V/V Methanol and water in the ratios of 50v/v and 50v/v.

2.4 Preparation of solutions

Standard Solution is prepared byweighing 50 mg of Promithazine Theoclate standard into a 100 ml standard flask. Added 5 ml water and 1 ml concentrated ammonia to the flask, mix, allow to stand for 5 minutes and make up to volume with absolute ethanol. Diluted 5 ml to 50 ml with absolute ethanol in an amber standard flask. Diluted a further 5 ml to 50 ml in standard flask and marked up to the mark by absolute ethanol. Sample solution is prepared by weighing 20 tablets, determined average mass and grind to a fine powder. Accurately weighed 500 mg of sample into a 100 ml standard flask. Added 5 ml water and 1 ml concentrated ammonia to the flask, mixed, allowed to stand for 5 minutes. Added 30 ml of absolute ethanol, sonicated for 5 minutes and allowed to cool. Maked up to volume with ethanol and filtered. Diluted 5 ml of this solution to 50 ml by absolute ethanol in 50 ml standard flask. Further diluted 5 ml to 50 ml in standard flask and make up to mark by absolute ethanol. Blank solution is prepared by diluting 0.05 ml concentrated ammonia to 50 ml standard flask with absolute ethanol. Further diluted 5 ml of this solution to 50 ml standard flask with absolute ethanol.

3. Method Development

A systematic study of the effect of various factors was undertaken by varying one parameter at a time and keeping all other conditions constant. Method development consists of selecting the appropriate wave length and choosing stationary and mobile phases. The following studies were conducted for this purpose. The spectrum of 10 ppm solution of Theophen Compound Elixirwas recorded separately on UV spectrophotometer. The peak of maximum absorbance wavelength 252nm was observed. Finally the expected separation and peak shapes were obtained on Jasco Model: V-630, with a fixed bandwidth 2 nm and a pair of 1cm matched quartz cell. Flow rates of the mobile phase were changed from 0.5-1.5 ml/min for optimum separation. It was found from experiments that 1.0ml/min flow rate was ideal for elution of analyte. The injection volume is estimated at 20 µl. Approx. Rt. for Theophylline Anhydrous is+ 12 minutes

4. Validation of the proposed method

4.1 Specificity

The Blank and placebo solution spectra must not show any absorbance corresponding in wavelength to that of the active compound. The spectra obtained from the drug active and sample must correspond. The blank and placebo does not to have any absorbance at specified wavelength corresponding to Promithazine Theoclate and drug active and product have corresponding absorbencies at about 203 nm, 255 nm and 276 nm. Therefore it is considered spectrally pure.The solvent and placebo solutions must contain no components, which co-elute with the Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous. The peak purity results from the photo diode-array analysis must show that the Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous peak is pure – i.e. the purity angle must be less than the threshold angle. The solutions listed below were injected using the conditions specified in the method of analysis. Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous stable are under UV light exposure. No components are seen to co-elute with the Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous peaks, and the peak purity results indicate that the Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous peaks can therefore be considered spectrally pure. The chromatograms are shown from figure 2 to 8.

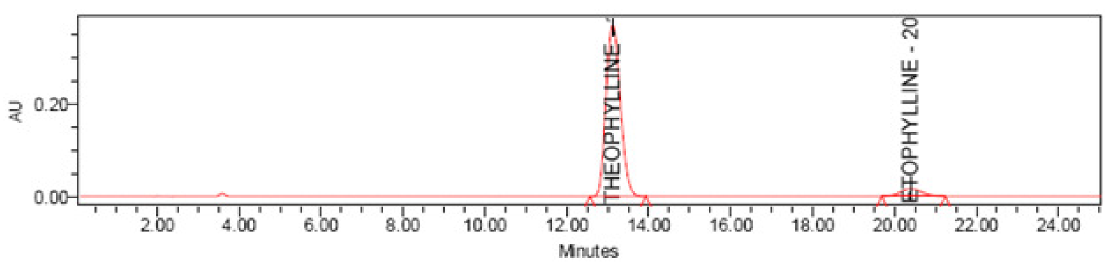

Fig.- 2:Chromatogram 1: Drug active – Peak due to Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous

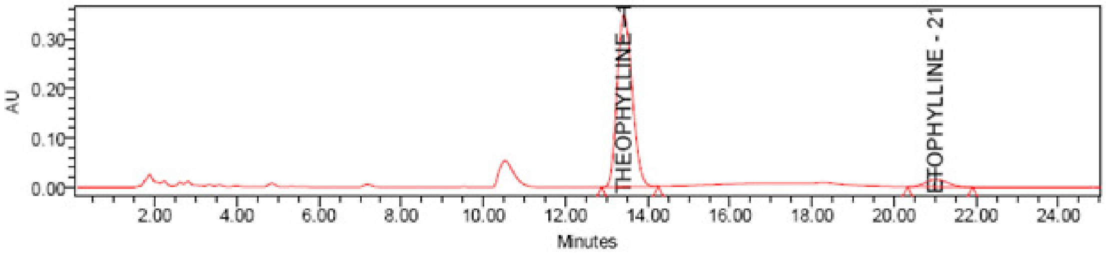

Fig.- 3:Chromatogram 2: Product – Peak due to Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous

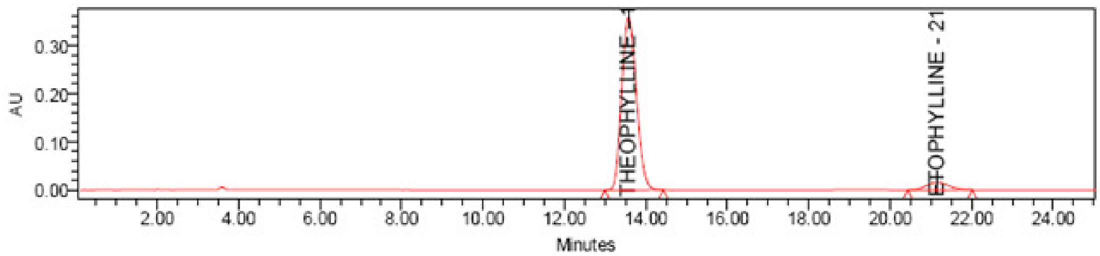

Fig.- 4:Chromatogram 3:Active – UV stress: Peak due to Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous

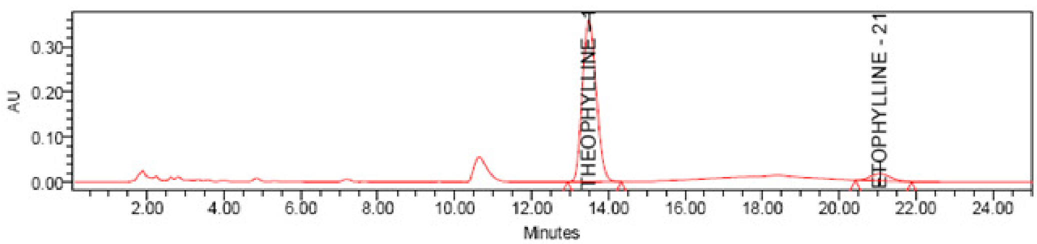

Fig.- 5:Chromatogram 4: Product – UV stress: Peak due Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous

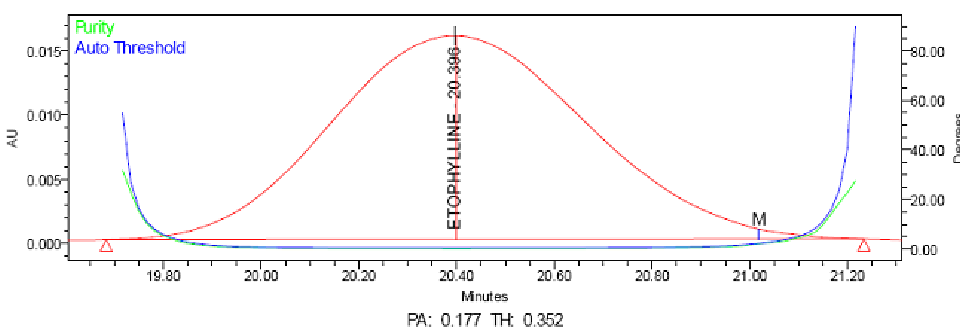

Fig.- 6: Peak Purity 1: Drug Active Etophylline (β Hydroxyethyl Theophylline)Purity Angle <Threshold: 0.177 < 0.352

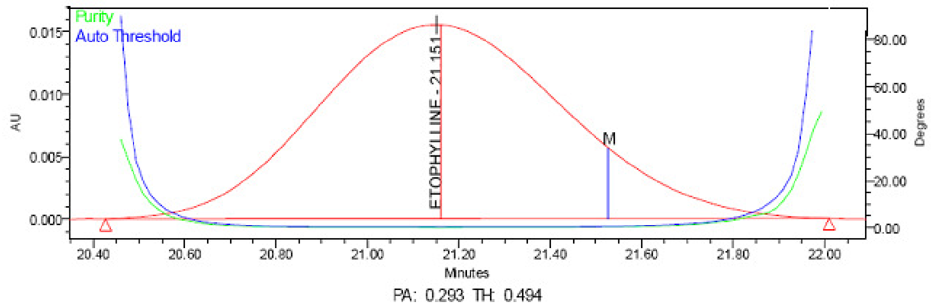

Fig.- 7. Peak Purity 2: Drug Active Etophylline (β Hydroxyethyl Theophylline)Purity Angle <Threshold: 0.293 < 0.494

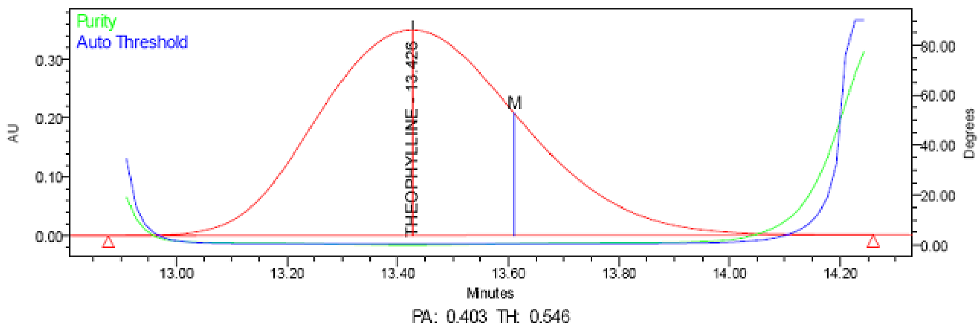

Fig.- 8: Peak Purity 3: Theophylline Peak Purity Angle <Threshold: 0.403 < 0.546

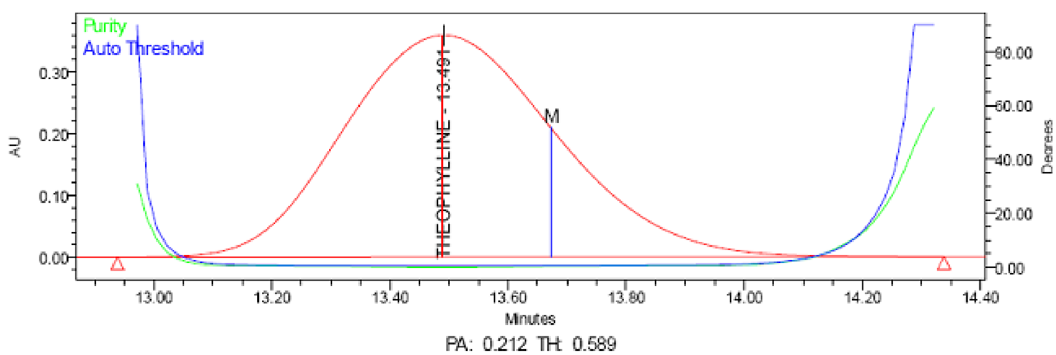

Fig.-9: Peak Purity 4: Theophylline UV Stressed Drug Product

4.2 System Suitability

The % RSD of the peak responses due to Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous for the six replicate injections for each must be less than or equal to 2.0 %. Six replicate injections of working standard solution were injected according to the method of analysis. The % RSD for the peak responses was determined. The analytical system complies with the requirements specified by the system suitability. The results are tabulated in the table 1.

Sample	Etophylline (β-Hydroxyethyl Theophylline)	Theophylline
1	583050	8322581
2	584320	8333629
3	584644	8339267
4	585002	8334340
5	585018	8329352
6	584723	8350223
Mean	584460	8334899
% RSD	0.1	0.1

Table1. System Suitability results

4.3 Linearity

The correlation coefficient of the regression line for Promethazine Theoclate should be greater than or equal to 0.99. The Y-intercept of the line should not be significantly different from zero, i.e. the assessment value (z) falls within the specified limits only when +5 > z > -5. Five solutions containing 50, 75, 100, 125, and 150 % of Promithazine Theoclate, relative to the working concentrations, were prepared and read according to the method of analysis. A linear regression curve was constructed, and the R^2 and assessment values calculated. The R^2 for Promithazine Theoclate is 1.00. The plot is a straight line, and the assessment value (z) is 1 for Promithazine Theoclate. Linearity Curve was shown in Figure 10 and results are in table 2.

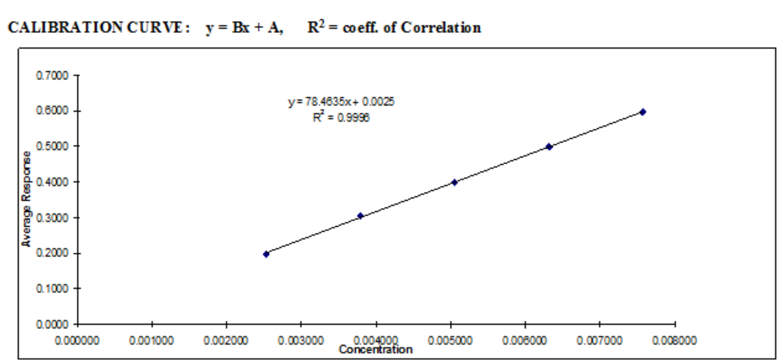

Fig.- 10. Linearity curve

Sample Number	Concentration	Response 1	Response 2	Average Response
50%	0.002528	0.1970	0.1970	0.1970
75%	0.003791	0.3050	0.3050	0.3050
100%	0.005055	0.3990	0.3990	0.3990
125%	0.006319	0.4990	0.4990	0.4990
150%	0.007573	0.5950	0.5950	0.5950

Table 2. Linearity results

4.4 Accuracy

The percentage recovery of the active compounds for each solution prepared are within 95.0 – 105.0 % of the actual amount. Sample solutions were spiked with known concentrations of Promethazine Theoclate to result in concentrations representing 50, 75, 100, 125, and 150 % of Promethazine Theoclate relative to the working concentration. The absorbance of the above samples was read in duplicate according to the method of analysis. From the accuracy results above, the percentage recovery values for Promethazine Theoclate satisfy the acceptance criteria for accuracy across the range of 50 - 150 %. Results are tabulated in table 3.

Sample	Theoretical Weight in mg	Response(Abs)	Average Response(Abs)	Average Actual Weight in mg	Average % Recovery
50%	24.31	0.189	0.189	23.90	98.3
50%		0.189			
75%	36.47	0.283	0.283	35.79	98.1
75%		0.283			
100%	48.61	0.379	0.379	47.93	98.6
100%		0.379			
125%	60.77	0.474	0.474	59.82	98.4
125%		0.473			
150%	72.92	0.563	0.563	71.20	97.6
150%		0.563			

Table 3. Accuracy Results

4.5 Method precision

4.5.1Repeatability

The % RSD due to Promethazine Theoclate concentration for the six samples must be less than or equal to 2.0 %. Six separate sample preparations of batch 241645 were analyzed according to the method of analysis. The % RSD due to Promethazine Theoclate concentration for the assay meets the requirements. Results are tabulated in table 4.

Sample number	**Results (mg/tab)**
	Promethazine Theoclate content
1	25.15
2	25.22
3	25.36
4	24.96
5	25.08
6	25.14
Mean	25.15
% RSD	0.5

Table 4. Repeatability results

4.5.2Intermediate Precision

The % RSD due to Promethazine Theoclate concentration for the six samples must be less than or equal to 2.0 %. The mean results obtained in the repeatability, and the intermediate precision must not differ by more than 3.0 %. Six separate sample preparations of batch 241645 were analysed according to the method of analysis. The % RSD for intermediate precision is 0.4 %.The intermediate precision and repeatability comply as they differ by 0.1 %. Results are tabulated in table 5.

Sample	**Results (mg/tab)**
	Promethazine Theoclate
1	24.99
2	25.05
3	25.09
4	25.18
5	25.16
6	25.24
Mean	25.12

% RSD	0.4
Sample	Mean Results (mg/tab)
	Promethazine Theoclate
Repeatability	25.15
Intermediate Precision	25.12
Mean	25.14
% RSD	0.1

Table 5: Intermediate precision results

4.5.3 Stability Of Solution

The assay results showed that no loss of active in the sample preparation while standing for time period of 24 hours. The % RSD due to the β-Hydroxyethyl Theophylline and Theophylline Anhydrous content for the samples is 1.5 %. The stability of solution mean of β-Hydroxyethyl Theophylline and Theophylline Anhydrous deviates from the obtained intermediate repeatability mean by 0.50 %.

4.5.4Range

Based on the accuracy results, the range for the assay of Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous is 5.0 – 15.0 mg/30ml and 50.0 – 150.0 mg/30 ml respectively, which represents 50 % to 150 % of the working concentration.

5. Resultsand Discussion

Several mobile phase compositions were tried. The chromatographic analysis was performed on a Jasco Model: V-630, with a fixed bandwidth 2 nm and a pair of 1cm matched quartz cell at ambient temperature. A satisfactory separation and good peak symmetry was found in a mixture of 0.05M KH_2PO_4 (to pH 3 with Phosphoric acid) 95V/V and Acetonitrile5v/v are used as a mobile phase. Flow rate proved to be better than the other mixtures in terms of resolution and peak shape. The wavelength for detection was set at 254nm at which much better detector responses for drug was obtained. The Rtfor β-Hydroxyethyl Theophylline and Theophylline Anhydrous is ± 12 minutes. A system suitability test was applied to representative chromatograms for various parameters. The results obtained were within acceptable limits. Therefore the system meets suitable criteria. The calibration curve was obtained for a series of concentration in the range of 0.6-1.4µg/ml and it was found to be linear. Five points graphs was constructed and identified that the correlation coefficient (R^2) for β-Hydroxyethyl Theophylline and Theophylline Anhydrous is 0.0007.The plot is a straight line and the assessment value (z) is 0.6 for β-Hydroxyethyl Theophylline and Theophylline Anhydrous. All these factors lead to the conclusion that the proposed method is accurate, precise, simple, sensitive and rapid and can be applied successfully. The Cleaning Validation method is proven to be valid and the validation test results show that the method complies with the validation requirements.The method is therefore acceptable as valid.

6. Conclusion

The method for the assay of Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous in Theophen Compound elixir complies with the requirements for linearity, specificity, system suitability, method precision and accuracy across the range of 50% to 150% of the working concentration. The method therefore indicates stability and is acceptable and valid.The developed method is also found to be precise and robust for the simultaneous determination of Promethazine hydrochloride and Theophylline Anhydrous in liquid dosage forms.

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Outcomes-Based Education: Planning, Teaching and Assessment Possibilities

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Abstract:

This paper discusses some of the basic principles of outcomes-based education, and addresses them in Indian Higher Education planning, teaching and evaluation. It will help teachers understand that OBE's theory and principles can be converted into concrete steps in the preparation, teaching and assessment of education for students. The paper acknowledges the fact that OBE has both opponents and supporters and quickly discusses some of the common concerns about OBE. In several ways, the paper is deliberately provocative and encouraging educators to challenge their existing teaching practices and explore ways of integrating those concepts in results-oriented learning. The article is not meant as a detailed OBE treatise. This presents several concepts that are central to OBE and suggests means of discussing these theories more closely.

Keywords: *Teachers' instructional planning, teaching and assessment of student learning.*

Introduction:

"A good archer is not known by his arrows but by his aim."

-Thomas Fuller

"A windmill is eternally at work to accomplish one end,
although it shifts with every variation of the weathercock,
and assumes ten different positions in a day."

-Charles C. Colton

Outcome-based education is easy to conceptualize but difficult to define. This is an education approach in which curriculum decisions are driven by the results that students must show at the end of the course. Education based on results can be summarized like' results-oriented thinking' and is the opposite of' input-based education,' which emphasizes the educational process and accepts whatsoever. The results agreed in outcomes-based education on what is being taught and assessed for the curriculum guide.

In the context of the learning outcomes it is clear what the education results will be and how they will be organized, the educational strategy, the teaching methods, the assessment procedures and the educational setting. At least three viewpoints are available to judge the quality of the educational system. The system inputs, what happens inside the system and the system outputs. Anyone willing to receive information should focus their attention mainly on money, capital, services, etc. and may base their opinion on the performance and usefulness of the process by using economic rationalism.

Those who are interested in results will focus mainly on products or educational outcomes. All aspects of education can be argued that are important and that quality is not to be assessed from a narrow perspective. However, there have been increased calls for more attention in western society in recent years to the outcomes of education in order to assess the return on educational investments (especially public education). The growing demand for accountability is one reason why diverse forms of outcome education spread rapidly over the 1980s and 1990s, in countries like the United States and the United Kingdom.

Each education system has two basic types of outcomes. The first type includes results such as results of tests, completion rates, employment after-course , etc. The second type of result is less measurable and generally shows the skill, the ability or the essence of the training of the learners. This is the second kind of result.

In Spady's words:

> " Outcome-Based Education means clearly focusing and organizing everything in an educational system around what is essential for all students to be able to do successfully at the end of their learning experiences." "This means starting with a clear picture of what is important for students to be able to do, then organizing the curriculum, instruction, and assessment to make sure this learning ultimately happens "
>
> (Spady, 1994:1).

Spady's main idea is that the OBE approach to planning, delivery and evaluation requires that administrators, teachers and students focus their attention and effort on the desired outcomes of education— the outcomes that are expressed by student learning in terms of individuals. One approach highlights the students ' mastery of traditional academic results (usually focused on the subject) and certain cross-disciplinary outcomes (e.g. the ability to solve problems or work collaboratively). The second approach focuses on long-term cross-curricular results directly related to the future roles of students (e.g. productive workers or responsible citizens or parents). These two approaches correspond to the traditional and transitional OBE and transformational OBE that Spady (1994) calls.

Spady clearly favours a revolutionary approach to OBE where "high quality, culminating demonstrations of significant learning in context" (Spady, 1994:18). For Spady, learning is not significant unless the results reflect the complexities of real life and promote the life roles faced by the students after their formal education has been finished.

OBE is guided by three basic assumptions in addition to the belief that the findings must reflect meaningful long-term learning:

- Every student can learn and excel, but not all at the same time.
- Successful learning encourages much more learning.
- Schools (and teachers) monitor the circumstances under which students will or will not be successful in school.

From his three premises, Spady developed four essential principles of OBE.

1. clarity of focus
2. designing back
3. high expectations
4. expanded opportunities

The first principle is clarity of focus: this means that all teachers must clearly focus on what they want students to do. In planning and teaching teachers should therefore focus on helping students to develop the knowledge, competences and arrangements that ultimately will allow them to achieve significant, clearly articulated results. This principle obliges teachers to make clear for the students at all stages of the teaching process their short-and long-term purpose for learning. It also needs educators to rely on clearly defined significant results for every student review.

The second principle is often described as a backrest and is inseparably bound up with the first principle. That means that a consistent description of the essential education that students should accomplish by the completion of their formal training must be made the starting point for all course development. All educational decisions are then based on the "desired final result" and the "building blocks" of learning that students must achieve so that the longer-term outcomes can eventually be achieved. Killen and Spady (1999) describes a systematic framework to develop curricula for higher education in this way, and Collier (2000) illustrates its application.

The third fundamental principle of OBE is that teachers should expect all students very much. In literature, there is ample evidence that teachers need to set the high and challenge level for performance to encourage students to get deeply involved in the issues they are learning from (e.g. Queenland School Reform Longitudinal Study 1999). The idea that successful learning promotes successful learning is very closely related to helping students achieve high standards (Spady 1994). As learners excel, they build their confidence, increase their faith, and embark on further learning challenges. One of the main reasons for using OBE is that it can motivate all students to do hard things properly.

Intellectual quality is not something for a few pupils: something that all pupils should expect, which is linked with the fourth principle: that pupils should endeavour to offer all pupils more opportunities. The belief that not all students learn the same way at the same time is based on this notion (Spady, 1994). However, most students can achieve high standards by offering adequate opportunities— the truth is that the students learn important things; they do not learn them in a specific manner or by a certain arbitrary means.

For example, we cannot conveniently ignore the design principle or the expanded opportunities principle and continue to say that we have an OBE system. One of the attractions of results-based education is the ability for managers to control the outcomes of education while at the same time offering teachers a great deal of freedom to select the content and methodologies, they use to help their students achieve the results. Both of these issues of control and freedom can create tension; teachers can struggle with authority and teachers can not like how teachers use their choice. Alternatively, it will try and show that the teachers can operate within a results-based system and have the ability to deal with a lot of social, moral and racial issues of teaching and learning at the same time. The paper will not settle the controversy because that challenge is tough.

Instructional Planning:

The OBE system has three main elements: deciding what students are to achieve, how students are to help them achieve these results (i.e., decide content and teaching strategy) and how students are to be determined when the students have reached the results (i.e., decide assessment and reporting procedure). The system includes three main steps in educational planning. Such decisions are made as a curriculum expert (e.g. a secondary science teacher) in the view of most students. However, if students are to achieve broader results–for example the key skills–they have to organize learning programs in an integrated fashion that draws on elements from all areas of learning.

Curriculum documents are broadly written in any State or in any national education system; therefore, they do not address the particular needs of individual schools or particular groups of students directly. Teachers need therefore sufficient detail for their daily work in order to be able to translate whatever course guidelines they receive into specific teaching curricula. The programs are therefore planning which guide the selection of lesson results, content, teaching strategies, resources and assessment procedures by each teacher. The programs may be for large work areas (e.g. four-year courses) or small work groups (e.g. a subject area). The nature of these programs will be very different, but they can be similar in structure. The objective of each program should be to clarify (to explain why it has been implemented), goal (to explain the successes of the program) outcomes statements, quality statements (to show the specific scope of the program), instructional method statements (to illustrate how the training experiences should be organized), and assessments. Every programming form addresses these questions at some point, but three basic styles of programming can be used by stressing different key elements.

1. Content-based programming:

The selection of content precedes the consideration of results or teaching strategies in content programming; content-based programming is the approach most teachers are familiar with. This stress "covers the course" almost entirely by implying that teachers must teach specified material over each period of time (lesson, word, year, etc.). In many cases, the content being taught is closely linked to a subject-based textbook. This approach takes little account of how much each student will learn in the time available and causes teachers to believe that it is acceptable and appropriate to learn different amounts of each individual student. However, we can find out that different differences are based on their learning ability. Yet teachers have not forgotten learner's outcomes.

2. *Activities-based* programming:

The choice of learning experiences takes precedence over other considerations in the activity-based programming; experience-based curriculum stresses student interactions by requiring students to think about what they could learn from these activities. It is very difficult to justify content or experience-based programming on a non-administrative basis.

3. *Outcomes-based* programming:

The first choice in outcomes-based programming is what students will learn about and be able to do when the course is done. "Scheduling results means organizing education to achieve predetermined results." It begins by specifying clearly what the students need to know, what they can do and what attitudes or values are desirable at the end of the programme.

> "In outcomes-based education . . . you develop the curriculum *from* the outcomes you want students to demonstrate, rather than writing objectives *for* the curriculum you already have."

(Spady, 1988:6).

The program is built to offer all students equal opportunities for each outcome, using these findings as a guide. The most important element of results-based education is that' all students are expected to be successful. There are many different means to approach results-based programming and evaluation.

In order for all the students to learn well and to achieve particular achievements, teachers must follow certain instructional processes and each has consequences for the plan and curriculum of their teachers.

- Teachers need to properly prepare their students to succeed. This requires teachers to understand exactly what students want to know, to anticipate difficulties and to minimize those difficulties.
- Teachers need to create a positive learning environment that helps students, no matter how easy or difficult the learning process is to find. In large part, your relationship with students depends on this positive environment, but it also depends on your effort to help the physical environment learn.
- Teachers need to help students understand why they need to learn (including what it will be of use to them in future) and how they will know when they learn it. Do not assume that students will notice the importance of what you teach because you know why. And never teach anything that is not useful to you.
- Teachers need to use a variety of teaching methods to help each student learn. We should not presume that all students are similarly capable of learning from one single education approach, and that no specific education method is a good way of helping students to achieve these learning outcomes. If you have taken the findings into account that students want to achieve, the content you are going to use to help students achieve these results, the characteristics of the students and the resources available, you have to select the best strategy. The so-called "student-centred" approaches should not always be considered to be the correct OBE technique to use. These are often adequate but more specific instructional approaches sometimes are acceptable (Killen, 1998).
- Teachers need to offer students sufficient opportunities to practice with their new knowledge and skills, so that they can explore, experiment, concrete errors and adapt their thinking under the guidance of the teacher. It is important to help students apply their new knowledge and skills instead of just building up new knowledge and skills.
- Teachers must help every student to personally each study episode (lesson or class of lessons) so that they are aware of what they have learned and where they are led. Don't assume that without your guidance students can do this.

TEACHING STRATEGIES

Teaching is only teaching when students learn. Therefore, “it remains the responsibility of educators to construct meaningful learning experiences that lead to the mastery of outcomes” (Cockburn, 1997:7). Teachers have to make informed decisions about teaching strategies to build meaningful learning experiences. There are often two basic teaching approaches: teacher-centred and student-centred.

1. ***Teacher-centred approaches:*** These are also referred to as explicit, deductive and expository instructional — examples of which are lectures or demonstrations. Through these teaching methods, the teacher determines what is to be understood and how the knowledge to be learned is delivered.
2. ***Student-centred approaches:*** These also referred to as learning through discovery, inductive learning, or learning from inquiries emphasize much more the role of the learners in the learning process, such as cooperative learning and research projects for students. You still have the education agenda when you use instructional methods focused on students, but you do have much less influence over what and how students learn. You are no longer a filter that needs to pass through all information before you reach the learners.

Based on how learning is organized, teaching strategies often include labels such as reading, class discussion, group work, cooperative learning, troubleshooting, student research, etc. Killen (1998) explains how these and several other techniques can be used in a broad range of instructional scenarios and points out that no technique is an appropriate method in which students can produce all learning results.

Whatever approach to teaching you use; it is important to keep the following points in mind:

- • LEARNING rather than teaching should be the main focus.
- • Impossible for students to learn unless they THINK.
- • The PROCESSS you use to include students in material and the CONTENT itself will promote and enable the analysis.
- • There is no distance on your subject – you need to support students to make LINKS in other topics.

• It is your job to help LEARN HOW TO LEARN students.

In the OBE system, you cannot assume that all students learn from a strategy such as a small group discussion equally well, and you cannot assume that every student in any fixed time will learn the same thing. If you are to assist all students in achieving the results of what you teach, you have to be flexible in the way you teach and the expectations you have at any time for each student. You have to accept that students will be in different stages in most courses and therefore work towards various short-term outcomes at the same time. You'll have to be creative, and you'll probably notice that you won't use full class teaching very often to support each pupil in your curriculum (under the restrictions of a traditional school system).

ASSESSMENT

It is only natural to think about how far individual students achieve these results when we focus our planning and instruction on the results of education, so that we can see to what extent our intentions have been met if you wish. So, we must think about the evaluation and this can also be a matter for certain teachers in an OBE system. Nonetheless, in theory good assessment practices in OBE do not vary from good evaluation practices in any other education system.

The four concepts that characterize OBEs have many significant consequences for the assessment of student learning. If the concepts of emphasis and development are followed, all assessments must be related to the essential long-term outcomes to be accomplished by students, or to the short-term effects that can emerge from these long-term outcomes. Focusing evaluation only on specific results that neglect the long-term intent of the program of research is clearly not appropriate. Of example, it is important to periodically review the progress of the students towards the accomplishment of core competences.

To be useful in an OBE system, assessment should conform to the following principles:

- The evaluation methods should be valid — in reality, they should evaluate what you intend to evaluate.
- Reliable testing methods – reliable findings should be produced.
- Assessment processes should be realistic — no insignificant considerations such as the cultural background of the applicant can affect them.
- Testing must demonstrate the knowledge and skills that students need to learn more.
- Testing should suggest something that educators and individual students do not learn. This means that students should be able to apply their knowledge and to extend them to the limit of their understanding.

- A thorough and clear assessment should be carried out.
- Assessment will promote the ability for every student to know important things;
- The evaluation should allow this individuality to be demonstrated because students are individuals.

At OBE, assessment should always help improve the education of students. For evaluation should help research, appraisal activities will actually provide students with opportunities to show what they have achieved and recognize what they still need to learn. Since learning is not just an addition of prior knowledge, it should help student's connective what they learn to their prior knowledge. Because this process is continuous restructuring.

The development of equity evaluations would require careful analysis not only of our assessments and the way we conduct them, but also of how the evaluation processes we use respond to different individuals and groups. The challenge is to develop evaluation tasks flexibly enough to give students a sense of achievement, challenge the upper reaches of understanding of the student and provide a window in the thinking of each student. In this way, students may need to allow several points of entry and exit for evaluation activities and to respond in ways that reflect different levels of knowledge and sophistication. Equity involves every student being able to learn the significant knowledge and qualifications that are assessed and the content that they have not been given an opportunity to learn cannot be assessed fairly.

Evaluations can help students learn important things only if they are based on standards that reflect high expectations of all students; assessment equity cannot be achieved as long as excellence is not required of all. If we want excellence, the expectation must be high enough to allow each student to learn important knowledge and skills with effort and good instruction. This realism is important, as Gardner (1991) indicates, if teachers know what students learn.

Learner's Responsibility for Learning:

In an OBE model, "learners are responsible for their own learning and success" are often proposed (Cockburn, 1997, p.6). The issue was probably caused by a misunderstanding of the theory behind the concept. This is of interest for teachers and parents. The theory accepts that education is essentially an individual and private activity, no matter what the educators do. For its students the teacher cannot learn; the teacher can make learning easier only. In this respect, OBE stresses the teacher's role to clearly defining the goals and helping students achieve the results. It also stresses the responsibility of the learner to try and achieve the results.

One of the challenges that students are accountable for their own education is that they can find it very difficult to know if they are learning or not. They may be able to see that they make mistakes or answer questions wrongly, but this doesn't necessarily mean they are aware that they don't learn. Missing performance may be equated rather than incomprehension to lack of effort. Many people have trouble understanding why they don't understand even when they know what it is, they must remember (Killen, Meade, Yli-Renko & Fraser, 1996). This gives the teachers a new responsibility to help students to diagnose and evaluate their approaches to learning. One of the drawbacks of outcomes-based education is that it helps students understand, be aware of what they really know and be aware of their own control over their own learnings.

Conclusion:

The principles of the OBE and the philosophies discussed above in this article are subject to four simple questions: What do we want students to learn? Why do we want students to learn these things? How can we best help students to learn these things? and, how will we know when students have learned? Such concepts can be seen as an interesting mix of philosophical positions, but are naturally most strongly rooted in logical empiricism. Spady's view on OBE is that we should not allow schooling (or other aspects of education and training) to be driven by an

> "Edu-centric paradigm—a paradigm defined by what the system is and (always) has been rather than by what it should and could be if student learning and future success in the Information Age were its true purpose and priority"
>
> (Spady, 1998:10).

In the assumption however, that OBE offers a text as a way to achieve things that are appropriate for all school contexts, OBE becomes ideologically fixed. Successful implementation of OBE would allow educators to contextualize the OBE concepts correctly. It should be clear from these principles that OBE is not an' event,' but a comprehensive approach to education. It is not a matter of "fitting the calendar" but of a set of ideas that affect the overall curriculum of the college. One of the reasons for successful student learning is that results-based education encourages teachers to be well-trained.

My answer to OBE's critics is generally to say: first understand, then try, then criticize. No learning system is perfect, and no program can "act," except for the dedication of educators. Clearly, OBE has not been the spectacular achievement its supporters have hoped would have been in certain other nations. Teachers should encourage students in their efforts to achieve significant and meaningful results to follow this approach to learning. Teachers will know that if all students succeed, and nobody engaged in the education should be satisfied by their efforts, they achieve their goals.

If teachers want to succeed with outcomes-based education, they need to adopt the position that "there is no such thing as failure, only feedback and results . . . success depends on how well we process the feedback we get regarding our efforts" (Alessi, 1991, p.14).

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Unsteady Hall effects on magneto hydrodynamics flow through a permeable Channel

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Abstract - In this paper we talk about the corridor current impact on the pulsatile stream of a thick incompressible liquid through a permeable medium in an adaptable channel affected by cross over attractive field. The non-straight conditions administering the stream are tackled utilizing annoyance method. Accepting long frequency estimate, the speed segments and weights on the divider are determined up to arrange in and the conduct of the pivotal and cross over speeds just as the anxieties is examined for various variety in the administering boundaries. The shear weights on the divider are determined all through the pattern of swaying at various focuses inside a frequency and the stream partition is dissected.

Keywords: MHD flows, hall current effects, porous medium, unsteady.

I. INTRODUCTION

MHD is unstable with the common association of a directing liquid stream and attractive field. The liquids being examined are electrically directing and nonmagnetic, which restricts them to hot ionized gases (plasmas) and solid electrolytes. Because of the immense utilizations of attractive fields related to rotational impacts in current material handling, for example, driving pivoting MHD generators, adjustment of slender attractive fluid movies, and homogeneity control of leading liquids. Liquid streams affected by an applied attractive field happen in certain designing cycles, similar to glass fabricating, raw petroleum refinement, polymer innovation, geothermal vitality extraction and limit layer control in the field of streamlined features and blood stream. As of late, hydro magnetic stream and warmth move in it have gotten extensive consideration because of their different applications in science, building, and industry.

Impacts of radiation, compound response and soret on insecure mhd free convective stream over a vertical permeable plate concentrated by Dharmaiah et al. [1]. Temperamental mhd convective warmth and mass exchange stream past a slanted moving surface with heat assimilation have been analyzed by Ramprasad et al. [2]. Synthetic response and soret impacts on casson mhd liquid over a vertical plate proposed by Charan et al. [3].Chemical response, radiation and dufour impacts on casson magneto hydro elements liquid stream over a vertical plate with heat source/sink have detailed by Vedavathi et al. [4]. Engineered reaction and radiation impacts on flimsy mhd free convective stream over a vertical penetrable plate have been clarified by Babyrani et al. [5]. Mhd free convective stream past a semi-endless vertical penetrable moving plate with heat assimilation examined by Balamurugan et al. [6]. Impact of slip condition on radiative mhd stream of a thick liquid in equal permeable plate direct in presence of warmth assimilation and synthetic response examined by Venkateswarlu et al. [7]. A temperamental magneto hydro dynamic warmth move stream in a turning equal plate channel through a permeable medium with radiation impact communicated by Dharmaiah [8]. Magneto hydro elements convective stream past a vertical permeable surface in slip-stream system tested by Dharmaiah et al. [9]. Impact of substance response on mhd casson liquid stream past a slanted surface with radiation explored by Dharmaiah et al. [10]. MHD transient free convection adjusted attractive and artificially receptive stream past a permeable slanted plate with radiation and temperature slope subordinate warmth source in slip stream system explored by Babyrani et al. [11]. The impact of compound response on warmth and mass exchange mhd stream ag, tio2 and cu water nano liquids over a semi unending surface examined by Dharmaiah et al. [12]. Impact of radiation retention, thick and joules scattering on mhd free convection artificially receptive and radiative stream in a moving slanted permeable plate with temperature subordinate warmth source closed by Balamurugan et al. [13]. Impact of radiation on warmth and mass exchange in mhd liquid stream over an interminable vertical permeable surface with concoction response determined by Babyrani et al. [14]. Investigation of warmth and mass exchange on mhd stream of nanofluid over a semi endless moving surface with dissemination thermo have been concentrated by Dharmaiah et al. [15]. Warmth move on mhd nanofluid stream over a semi vast level plate implanted in a permeable medium with radiation retention, heat source and dispersion thermo impact created by Vedavathi et al. [16]. Examination of warmth and mass exchange on mhd stream with ag, al2o3 and cu water nanofluids over a semi limitless

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The Hall flows significantly affect the greatness and course of the current thickness and thusly on the attractive power. In practically the entirety of the previously mentioned examinations, steady liquid properties were accepted. Nonetheless, tests demonstrate this can hold just if the temperature doesn't change quickly or rashly. Thus, more exact expectation of stream and warmth move can be gotten distinctly by thinking about varieties of the liquid and electromagnetic properties, particularly the temperature varieties of the liquid thickness, warm conductivity, and the electrical conductivity. As a rule, most greases utilized in both building and mechanical cycles are receptive, e.g., hydrocarbon oils, engineered esters and so forth., and their effectiveness relies to a great extent upon the temperature variety. Accordingly, it is essential to decide the warmth move conditions and warm stacking properties of thick responsive liquids to gauge their viability as greases. Our paper is given to exploring the impacts of the Hall flows on a shaky hydromagnetic stream. The non-direct conditions overseeing the stream are illuminated utilizing irritation procedure.

II. FORMULATION AND SOLUTION OF THE PROBLEM

Consider the rickety fully developed pulsatile flow of fluid through a porous medium in a flexible channel of transverse field of magnetic strength H_O.

- Choosing the O(*x*, *y*), upper and lower walls of the channel are given by $y = \pm\, as\left(\frac{x}{\lambda}\right)$.
- The entire flow is subjected to strong uniform transverse magnetic field normal to the plate in its own plane.
- Equation of motion in both directions $\mu_e J_y H_o$ and $-\mu_e J_x H_o$.

The equations governing using Brinkman's model are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\mu_e J_y H_0-\frac{\nu}{k}u \qquad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\mu_e J_x H_0-\frac{\nu}{k}v \qquad (3)$$

$$J+\frac{\omega_e\tau_e}{H_0}J\times H=\sigma\,(E+\mu_e\, q\times H) \qquad (4)$$

In equation (4), the electron pressure gradient, the ion-slip and thermo-electric effects are neglected. We also assume that the electric field $E=0$ under assumptions reduces to

$$J_x+m\,J_y=\sigma\mu_e H_0 v \qquad (5)$$

$$J_y-m\,J_x=-\sigma\mu_e H_0 u \qquad (6)$$

where $m=\omega_e\tau_e$ is the hall parameter.

On solving equations (5) and (6) we obtain

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$$J_x = \frac{\sigma\mu_e H_0}{1+m^2}(v+mu) \tag{7}$$

$$J_y = \frac{\sigma\mu_e H_0}{1+m^2}(mv-u) \tag{8}$$

Using the equations (7) and (8) the equations of the motion with reference to frame are given by

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{\sigma\mu_e{}^2 H_0{}^2}{\rho(1+m^2)}(mv-u)-\frac{v}{k}u \tag{9}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+v\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\sigma\mu_e{}^2 H_0{}^2}{\rho(1+m^2)}(v+mu)-\frac{v}{k}v \tag{10}$$

Eliminating p from equations (9) and (10), the governing the flow in terms of appropriate stream function ψ reduces to

$$(\nabla^2\psi)_t-\psi_y\nabla^2\psi_x+\psi_x\nabla^2\psi_y=-\left(\frac{\sigma\,\mu_e^2 H_0^2}{\rho(1+m^2)}+\frac{v}{k}\right)\nabla^2\psi+v\,\nabla^4\psi \tag{11}$$

Where ∇^2 is the laplacian operator

The relevant conditions on ψ are

$$u=-\frac{\partial\psi}{\partial y},v=\frac{\partial\psi}{\partial x} \tag{12}$$

The relevant boundary conditions are

$$\psi=0,\ \psi_{yy}=0 \qquad \text{on } y=0 \tag{13}$$

$$\psi_y=0,\ \psi=1+k_1e^{it} \qquad \text{on } y=s \tag{14}$$

We introduce the following non-dimensional variables.

$$x^*=\frac{\psi}{\lambda},y^*=\frac{y}{a},t^*=\omega t,\ \varepsilon=\frac{a}{\lambda},\psi^*=\frac{\psi}{q\,a},\psi^*{}_f=\frac{\psi_f}{q_a}$$

Substituting the above non-dimensional variables into the equation (11), the governing equation in terms of non-dimensional parameter ψ (on dropping the asterisks) reduces to

$$R\varepsilon\left(\varepsilon^2(\psi_x\psi_{yxx}-\psi_y\psi_{xxx})+\psi_x\psi_{yyy}-\psi_y\psi_{xyy}\right)+S\varepsilon^2\psi_{tyy}+S\psi^4\psi_{txx}=$$

$$\varepsilon^4\psi_{xxxx}+\psi_{yyyy}+\varepsilon^2\left[2\psi_{xxyy}-\left(\frac{M^2}{1+m^2}+D^{-1}\right)\psi_{xx}\right]-\left(\frac{M^2}{1+m^2}+D^{-1}\right)\psi_{yy} \tag{15}$$

Equation (15) is highly non-linear and is not amenable for exact solution. However assuming the slope of the flexible channel ε small (<<1). We take ψ may be given asymptotic expansion in the form

$$\psi=\left(\psi_0 k_1 e^{it}\,\bar{\psi}_0\right)+\varepsilon\left(\psi_1+k_1e^{it}\,\bar{\psi}_1\right)+------- \tag{16}$$

We are making use of transformation

$$\eta=\frac{y}{s(x)} \tag{17}$$

And the boundary conditions at $y = s(x)$, Now to be satisfied at $\eta=1$.

The solution of the problem is given by

$$\psi=[C_1\sinh(\sigma_1\eta)+C_2\eta]+k_1e^{it}[C_8\sinh(\sigma_1\eta)+C_9\eta]+\varepsilon\{\ C_3\sinh(\sigma_1\eta)+c_4\eta+$$
$$C_5\eta^2\sinh(\sigma_1\eta)+C_6\eta\cosh(\sigma_1\eta)+C_7\sinh(2\sigma_1\eta)+k_1e^{it}(\ C_{10}\sinh(\sigma_1\eta)+$$
$$+C_{11}\eta+C_{12}\eta^2\sinh(\sigma_1\eta)+C_{13}\eta\cosh(\sigma_1\eta)+C_{14}\sinh(2\sigma_1\eta)\ )\ \} \tag{18}$$

The axial velocity and the transverse velocity are given by

$$u=\frac{-1}{s(x)}\{\ (C_1\sinh(\sigma_1\eta)+C_2\eta)_\eta+k_1e^{it}(C_8\sinh(\sigma_1\eta)+C_9\eta)_\eta+$$

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$$+\varepsilon\left(\left[(C_3\sinh(\sigma_1\eta)+C_4\eta+C_5\eta^2\sinh(\sigma_1\eta)+C_6\eta\cosh(\sigma_1\eta)+C_7\sinh(2\sigma_1\eta)\right]_\eta+\right.$$

$$\left.+k_1e^{it}\left[C_{10}\sinh(\sigma_1\eta)+C_{11}\eta+C_{12}\eta^2\sinh(\sigma_1\eta)+C_{13}\eta\cosh(\sigma_1\eta)+C_{14}\sinh(2\sigma_1\eta)\right]_\eta\right)\}$$

$$v=(C_1\sinh(\sigma_1\eta)+C_2\eta)_x+k_1e^{it}(C_8\sinh(\sigma_1\eta)+C_9\eta)_x+$$

$$+\varepsilon\{\left[(C_3\sinh(\sigma_1\eta)+C_4\eta+C_5\eta^2\sinh(\sigma_1\eta)+C_6\eta\cosh(\sigma_1\eta)+C_7\sinh(2\sigma_1\eta)\right]_x+$$

$$+k_1e^{it}\left[C_{10}\sinh(\sigma_1\eta)+C_{11}\eta+C_{12}\eta^2\sinh(\sigma_1\eta)+C_{13}\eta\cosh(\sigma_1\eta)+C_{14}\sinh(2\sigma_1\eta)\right]_x\}$$

Shear stress at the wall $y=s(x)$ is given by

$$\tau=\frac{\sigma_{xy}(1-s_{xx})+(\sigma_{yy}-\sigma_{xx})s_x}{1+(s_x)^2}$$

Where $\sigma_{xy}=-\mu(\psi_{yy}-\psi_{xx})$, $s(x)=1+\delta\sin x$, $\sigma_{yy}-\sigma_{xx}=1-4\mu\psi_{xy}$

III. RESULTS AND DISCUSSION

The stream administered by the non-dimensional boundaries R the Reynolds number, D^{-1} converse Darcy boundary, the abundance of the limit wave, k_1 the plentifulness of oscillatory transition, M the attractive boundary (Hartman number), S the oscillatory boundary and m lobby boundary. The pivotal, cross over speeds and the burdens are assessed computationally for various varieties in the administering boundaries R, D^{-1}, k_1, M, S and m. For computational reason we picked the limit wave s(x) = 1+ sinx in the non-dimensional structure. The figures (1-15) speak to the speed parts u and v for various varieties of the overseeing boundaries being different boundaries fixed. We see that for all varieties in the overseeing boundaries, the hub speed u accomplishes its most extreme on the mid plane of the channel. We notice that the greatness of the hub speed u improves and the cross over speed v lessens with expanding the Reynolds boundary R. The conduct of cross over speed v is oscillatory with its greatness diminishing on R increments through little qualities fewer than 30 and later decreases for additional expansion in R. The resultant speed likewise improves with expanding the Reynolds boundary R (Fig 1 and 2). From figures (3 and 4) we inferred that both the speed parts u and v diminishes with increment in the reverse Darcy boundary D^{-1}. Here we see that higher the porousness of the permeable medium bigger the hub speed along the channel and pace of increment is adequately high. Essentially, the resultant speed diminishes with expanding in the opposite Darcy boundary D^{-1}. It is obvious that the greatness of u, v and the resultant speed increment with expanding the boundaries k1, and m (5, 6, 9, 10, 13 and 14). The extent of the hub speed u upgrades and the cross over speed v diminishes with expanding in the abundance of the limit wave. The resultant speed additionally lessens with expanding the boundary (Fig 7 and 8). We notice that the greatness of the speed segments u and v lessens with expanding the power of the attractive field M. The resultant speed additionally diminishes with expanding the Hartmann number M (Fig 11 and 12). The size of the pivotal speed u upgrades and the cross over speed v decreases with expanding the oscillatory boundary S. The conduct of cross over speed v is oscillatory with its size diminishing on S increments through little qualities fewer than 3 and later decreases for additional expansion in S. The resultant speed likewise upgrades with expanding the oscillatory boundary S (Fig 15).

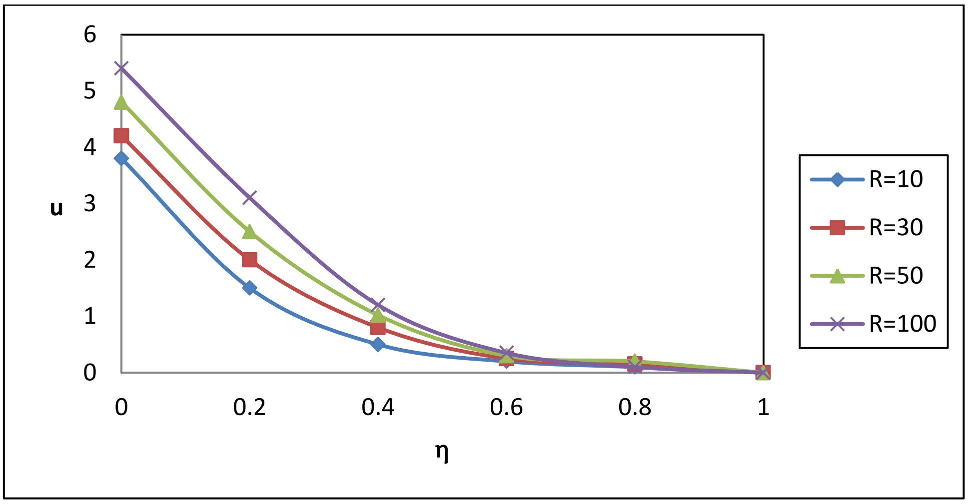

Fig 1: The velocity profile for *u* against *R*

k_1=1, S=1, D^{-1} =1000, M=2, m=1, $x=t=\frac{\pi}{4}$, $\delta=0.01$

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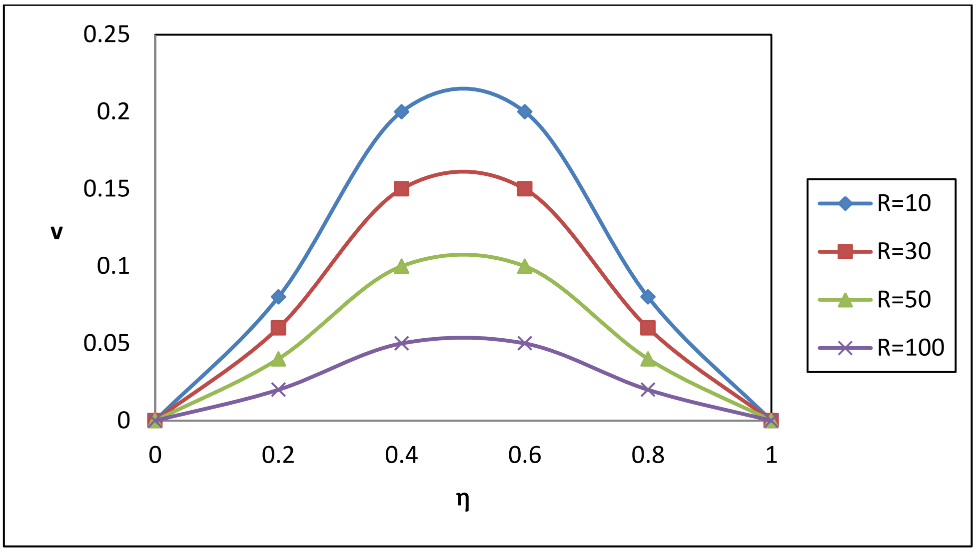

Fig 2: The velocity profile for *v* against *R*

k_1=1, *S*=1, D^{-1} =1000, *M*=2, *m*=1, $x = t = \dfrac{\pi}{4}$, $\delta = 0.01$

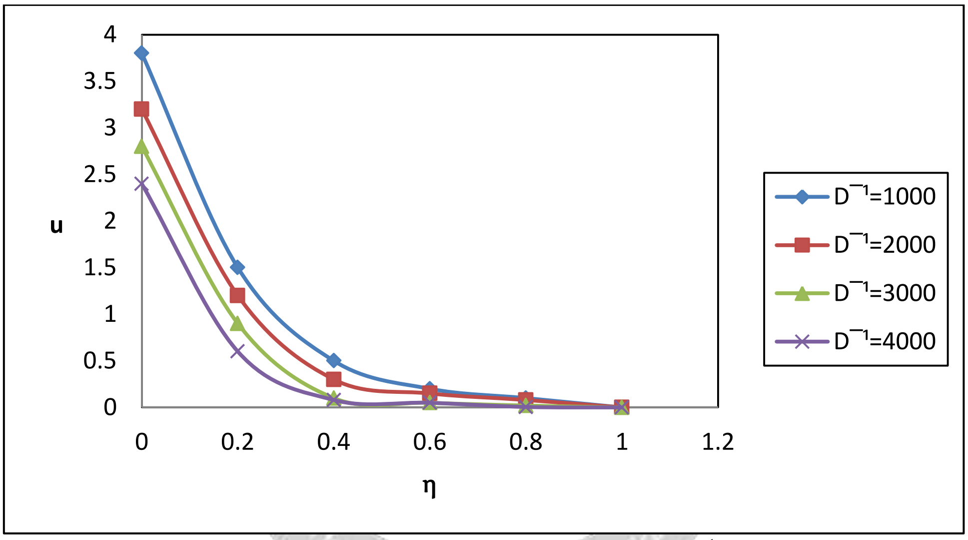

Fig 3: The velocity profile for *u* against D^{-1}

R=10, *S*=1, k_1=1, *M*=2, *m*=1 $x = t = \dfrac{\pi}{4}$, $\delta = 0.01$

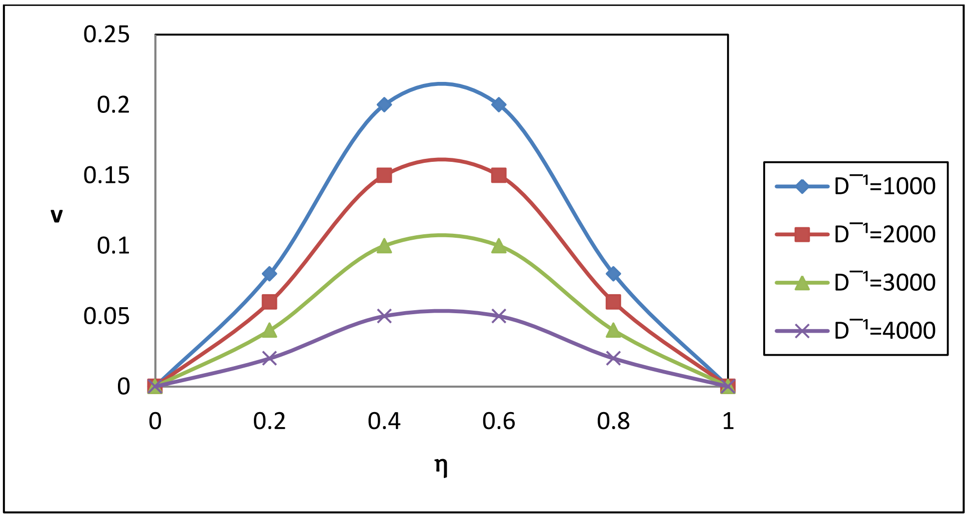

Fig 4: The velocity profile for *v* against D^{-1}

R=10, *S*=1, k_1=1, *M*=2, *m*=1 $x = t = \dfrac{\pi}{4}$, $\delta = 0.01$

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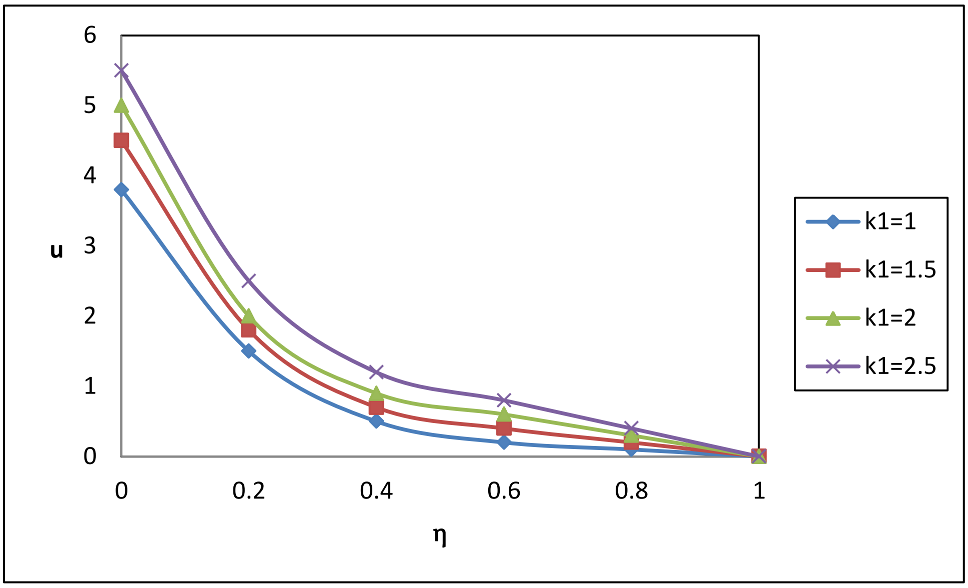

Fig 5: The velocity profile for u against k_1

m=1, S=1, D^{-1} =1000, M=2, $x = t = \dfrac{\pi}{4}$, R=10, $\delta = 0.01$

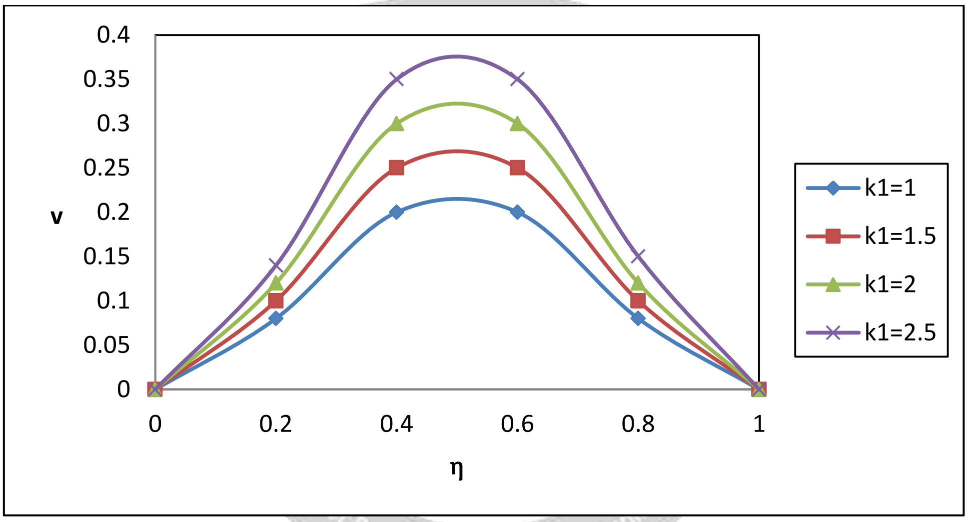

Fig 6: The velocity profile for v against k_1

m=1, S=1, D^{-1} =1000, M=2, $x = t = \dfrac{\pi}{4}$, R=10, $\delta = 0.01$

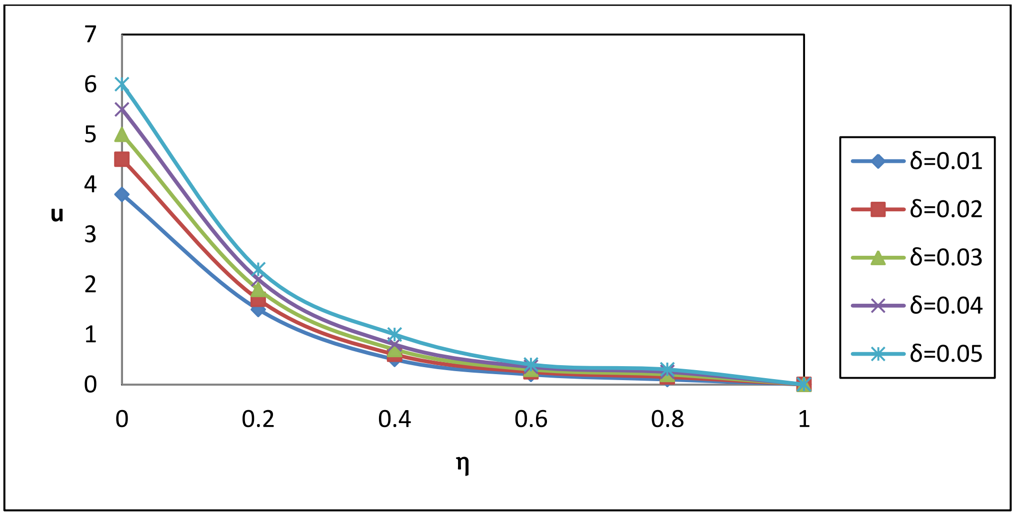

Fig 7: The velocity profile for u against δ

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k_1=1, S=1, D^{-1} =1000, R=10, M=2, $x = t = \dfrac{\pi}{4}$, m=1

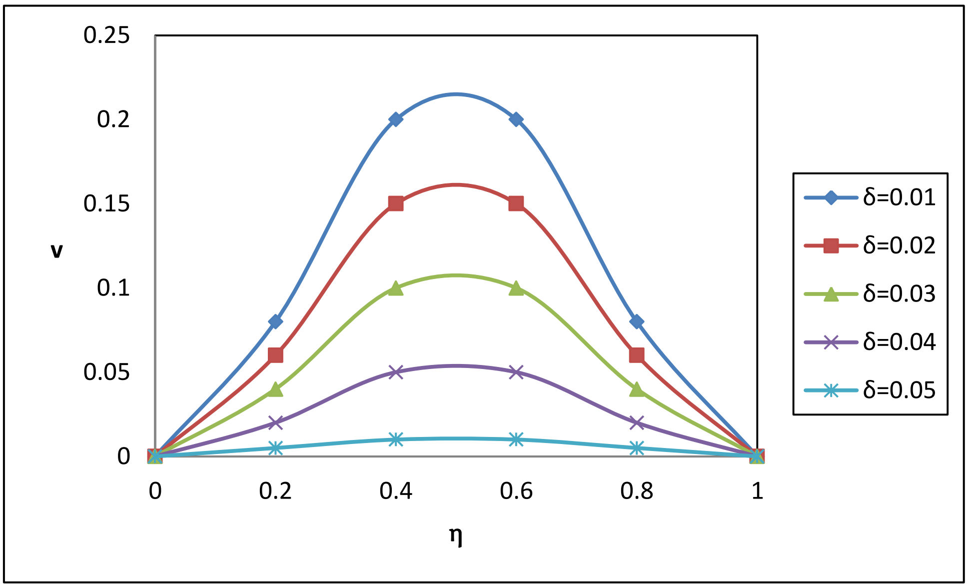

Fig 8: The velocity profile for v against δ

k_1=1, S=1, D^{-1} =1000, R=10, M=2, $x = t = \dfrac{\pi}{4}$, m=1

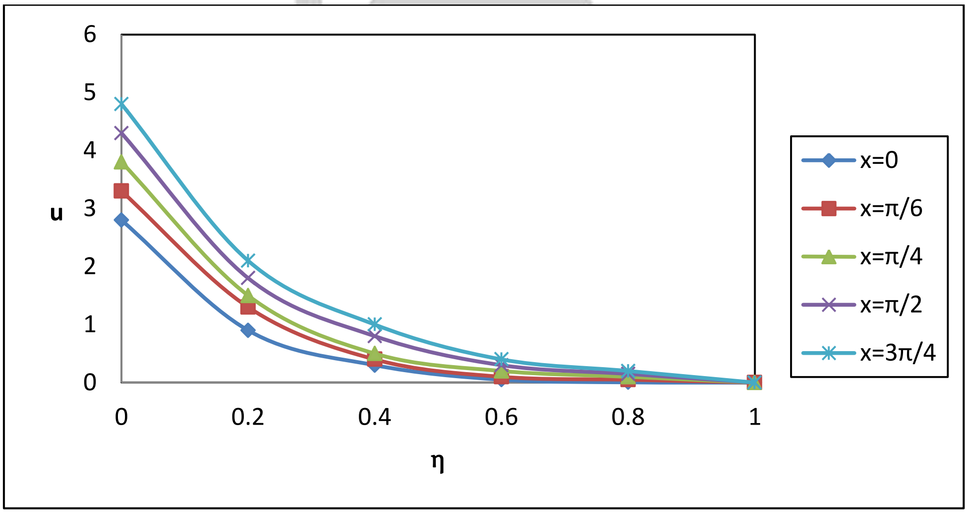

Fig 9: The velocity profile for u against x

R=10, S=1, D^{-1} =1000, k_1=1, M=2, m=1, $t = \dfrac{\pi}{4}$, $\delta = 0.01$

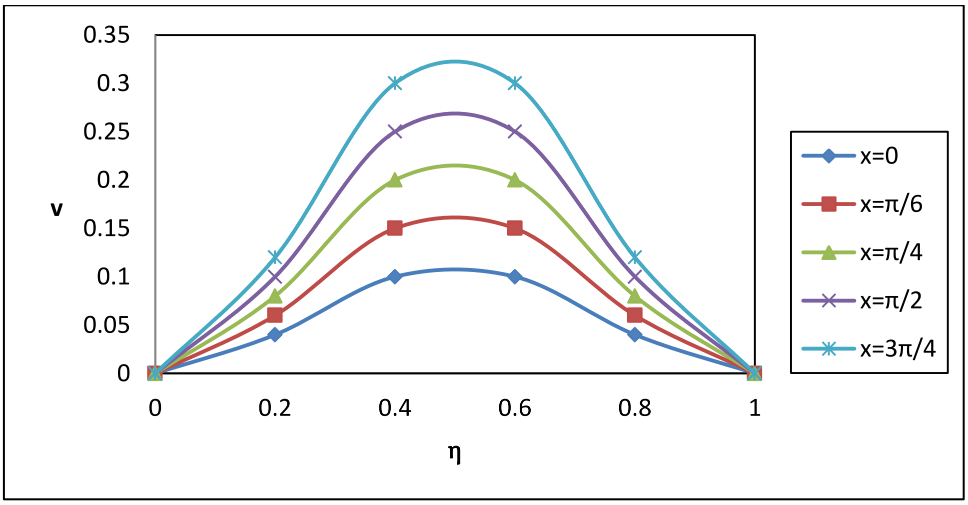

Fig 10: The velocity profile for v against x

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$$R=10,\ S=1,\ D^{-1}=1000,\ k_1=1,\ M=2,\ m=1,\ t=\frac{\pi}{4},\ \delta=0.01$$

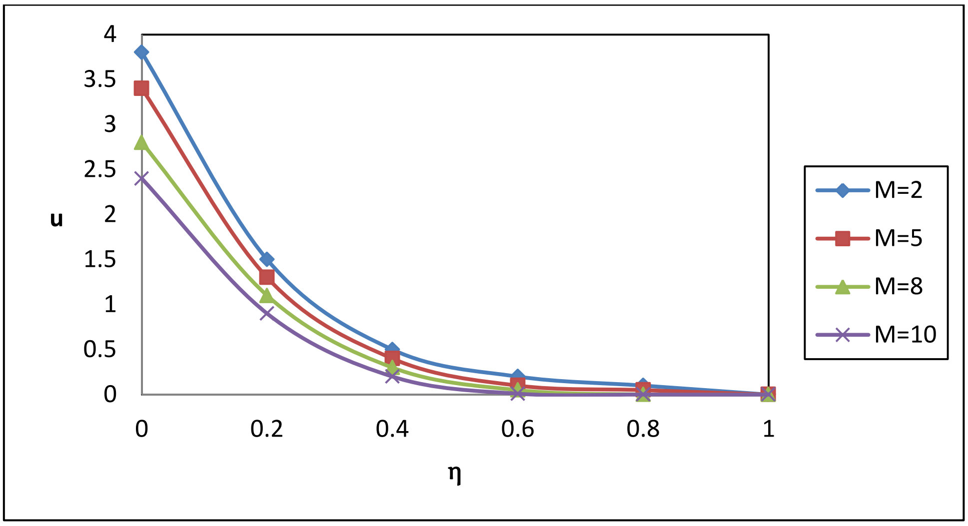

Fig 11: The velocity profile for *u* against M

$$k_1=1,\ S=1,\ D^{-1}=1000,\ m=1,\ x=t=\frac{\pi}{4},\ R=10,\ \delta=0.01$$

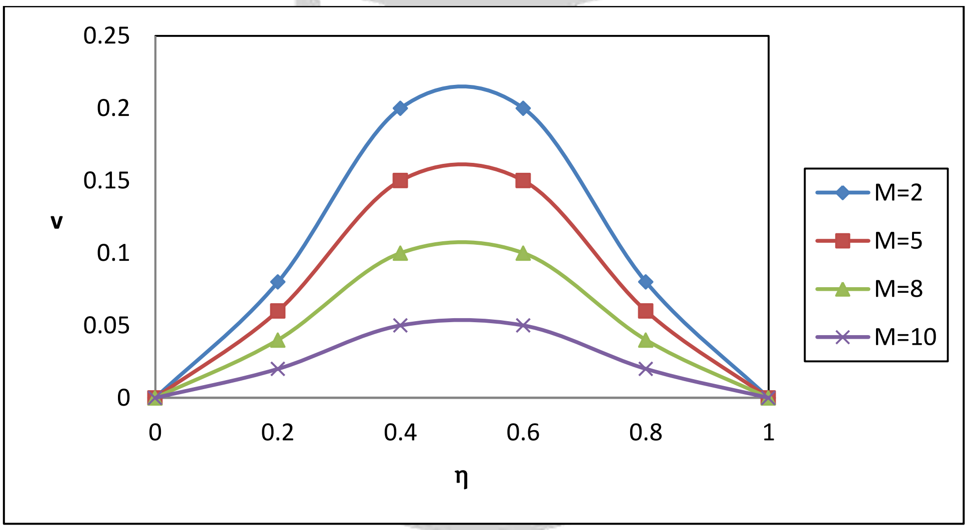

Fig 12: The velocity profile for *v* against M

$$k_1=1,\ S=1,\ D^{-1}=1000,\ m=1,\ x=t=\frac{\pi}{4},\ R=10,\ \delta=0.01$$

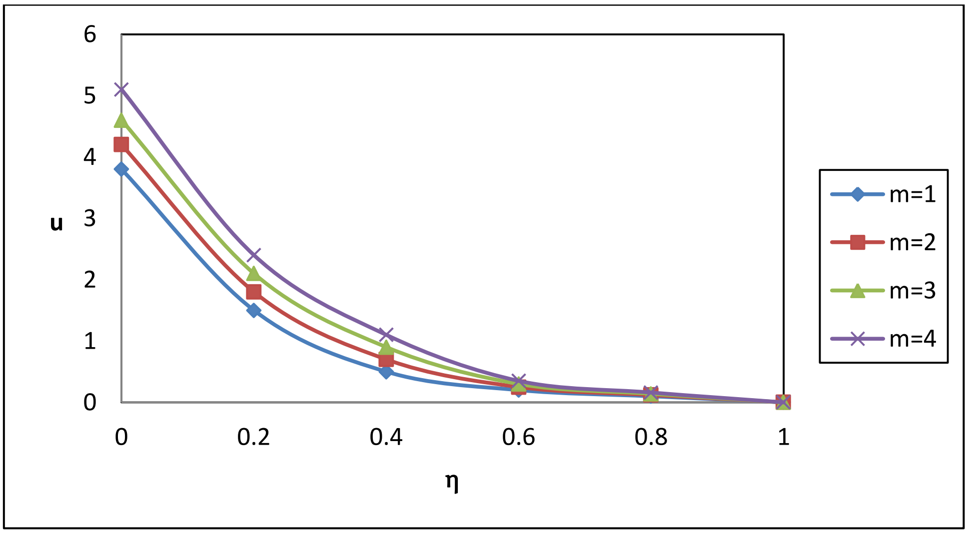

Fig 13: The velocity profile for *u* against *m*

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k_1=1, S=1, D^{-1} =1000, M=2, $x = t = \dfrac{\pi}{4}$, R=10, $\delta = 0.01$

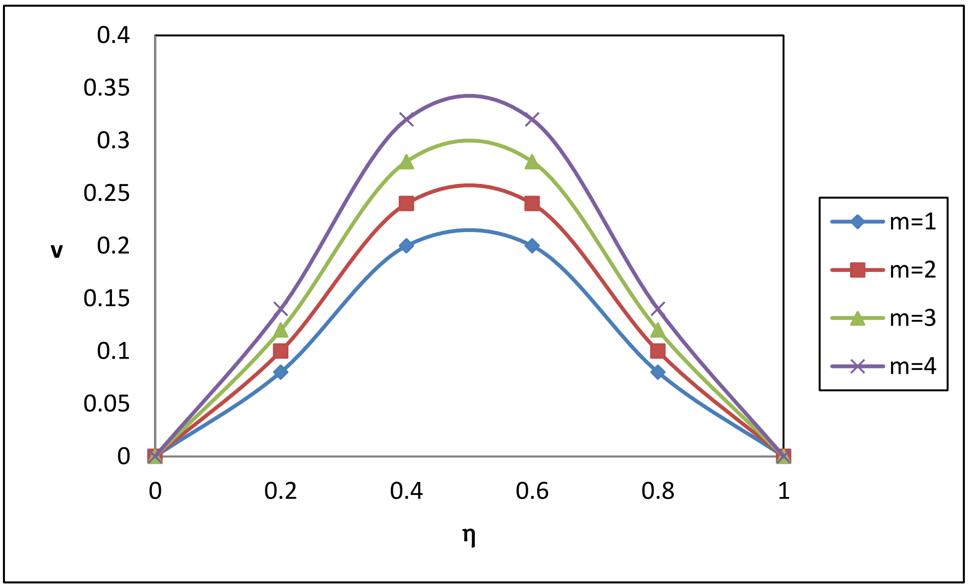

Fig 14: The velocity profile for *v* against *m*

k_1=1, S=1, D^{-1} =1000, M=2, $x = t = \dfrac{\pi}{4}$, R=10, $\delta = 0.01$

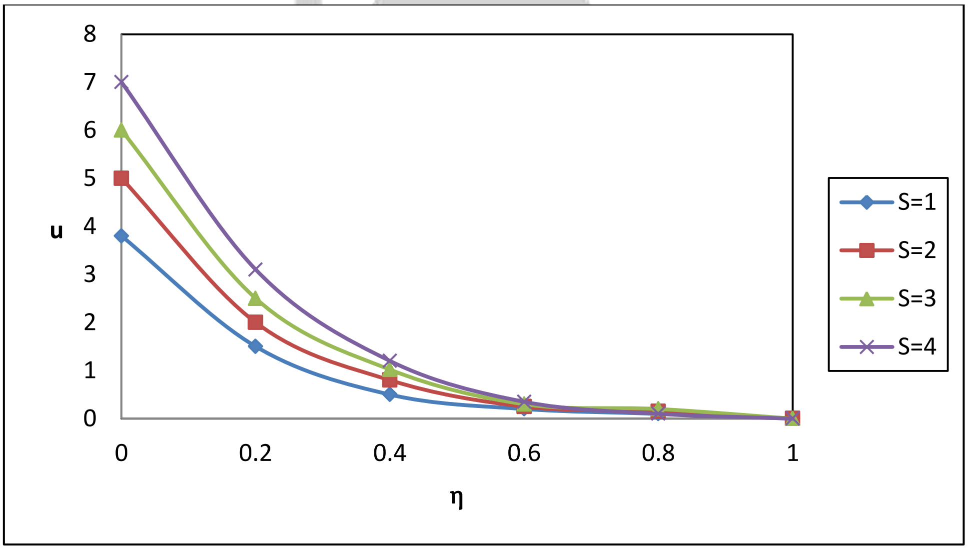

Fig 15: The velocity profile for *u* against *S*

k_1=1, D^{-1} =1000, M=2, m=1, $x = t = \dfrac{\pi}{4}$, R=10, $\delta = 0.01$

IV. CONCLUSIONS

The resultant speed likewise improves with expanding the Reynolds boundary R. Higher the penetrability of the permeable medium bigger the pivotal speed along the channel and pace of increment is adequately high. The resultant speed decreases with expanding in the opposite Darcy boundary D-1. The resultant speed increment with expanding the boundaries k_1 and m. The resultant speed additionally lessens with expanding the boundary .The resultant speed likewise diminishes with expanding the Hartmann number M. The size of the hub speed u upgrades and the cross over speed v lessens with expanding the oscillatory boundary S. The resultant speed additionally upgrades with expanding the oscillatory boundary S.

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Marshall-Olkin Stereographic Circular Logistic Distribution

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Abstract

Marshall and Olkin (1997) proposed an interesting method of adding a new parameter to the existing distributions. The resulting distributions are called the Marshall-Olkin distributions, these distributions include the original distributions as a special case and are more flexible and represent a wide range of behavior than the original distributions. In this paper, a new class of asymmetric stereographic circular logistic distribution is introduced by using Marshall-Olkin transformation on stereographic circular logistic distribution (Dattatreyarao et al (2016)), named as Marshall-Olkin Stereographic Circular Logistic Distribution. The proposed model admits closed form density and distribution functions, generalizes the stereographic circular logistic model and is more flexible to model various types of data (symmetric and skew-symmetric circular data).

Keywords:*Characteristics**,** Stereographic circular logistic distribution, circular data, Marshall-Olkin transformation, l-axial data.*

1. Introduction

Directions in two-dimensions can be represented as points on the circumference of a unit circle and models for representing such data are called circular distributions. There are many areas that deals with directional/circular data such as animal, birds navigation, sound waves in physics, wind direction in meteorology, image analysis in computer science and vector cardiology in medical science, to name some of them. For modeling circular data, Dattatreyarao et al (2007), Jammalamadaka and SenGupta (2001)etc derived many models for such data. Most of these models are typically symmetric around some center and very few asymmetricaldistributions are available for describing circular data. In this paper, we describe a broad class of model for the asymmetric case and discuss anapplication.

Phani (2013) constructed new circular models, Stereographic Circular Logistic, Stereographic Extreme-Value, Stereographic Double Exponential and some Stereographic Semicircular models by applying Inverse Stereographic Projection on linear models.Adding parameters to a well-established distribution is a time honored device for obtaining more flexible new families of distributions. Marshall and Olkin(1997) proposed an interesting method of adding a new parameter to the existing distributions. The resulting distributions, called the Marshall-Olkin (MO) distributions, which includes the parent distributions as a special case and gives more flexible to model various types of data. In this paper, we use the Marshall-Olkin transformation to derive a new flexible circular model, so called The Marshall-Olkin Stereographic Circular Logistic Distribution, which generalizes the baseline distribution (i.e., Stereographic Circular Logistic Distribution) for modeling circular data.

2. Methodology of Marshall-Olkin Transformation

An ingenious general method of adding a parameter to a family of distribution is introduced by Marshall-Olkin (1997). Here we adapt the idea of Marshall-Olkin to circular case for deriving more flexible circular models. For a circular random variable θ with a distribution function $F(\theta)$, we can obtain a new family $G(\theta)$ which contains one more parameter is given by $$G(\theta)=\frac{F(\theta)}{\alpha+(1-\alpha)F(\theta)},\ \theta\in[-\pi,\pi)\ , \qquad (1)$$

Where $F(\theta)$ is a distribution function and $\alpha>0$ .If $\alpha=1$ then we have $G(\theta)=F(\theta)$ and the probability density function corresponding to (2.1), say $g(\theta)$ takes the form
$$g(\theta)=\frac{\alpha f(\theta)}{\left(1-(1-\alpha)(1-F(\theta))\right)^2},\ -\pi\le\theta<\pi. \qquad (2)$$

Where $F^{'}(\theta)=g(\theta)$ is the baseline density function.

3. Marshall–Olkin Stereographic Circular Logistic Distribution

Here probability density and distribution functions of Stereographic Circular Logistic Distribution are revisited Phani (2013).

Stereographic Circular Logistic Distribution

A random variable θ on unit circle is said to have stereographic circular logistic distribution with location parameter μ and scale parameter $\sigma>0$ denoted by SCLG(μ,σ) , if the probability density and cumulative distribution functions are respectively given by

$$f(\theta)=\frac{1}{2\sigma}\sec^2\left(\frac{\theta}{2}\right)\left[1+\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-2}\exp\left(-\left(\frac{\tan\left(\frac{\theta}{2}\right)-\mu}{\sigma}\right)\right),$$

Where $\sigma>0$ and $-\pi\le\theta,\mu<\pi$

$$F(\theta)=\left[1+\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-1},\text{where } \sigma>0,\ -\pi\le\theta<\pi.$$

Then by applying Marshall-Olkin transformation on stereographic circular logistic distribution, we obtain more flexible asymmetric circular distribution, we call it as Marshall-Olkin Stereographic Circular Logistic Distribution.

Definition:

A random variable θ on unit circle is said to have Marshall-Olkin stereographic circular logistic distribution with location parameter μ, scale parameter $\sigma > 0$ and tilt parameter $\alpha > 0$ denoted by MOESCLG(μ, σ, α), if the probability density and cumulative distribution functions are respectively given by

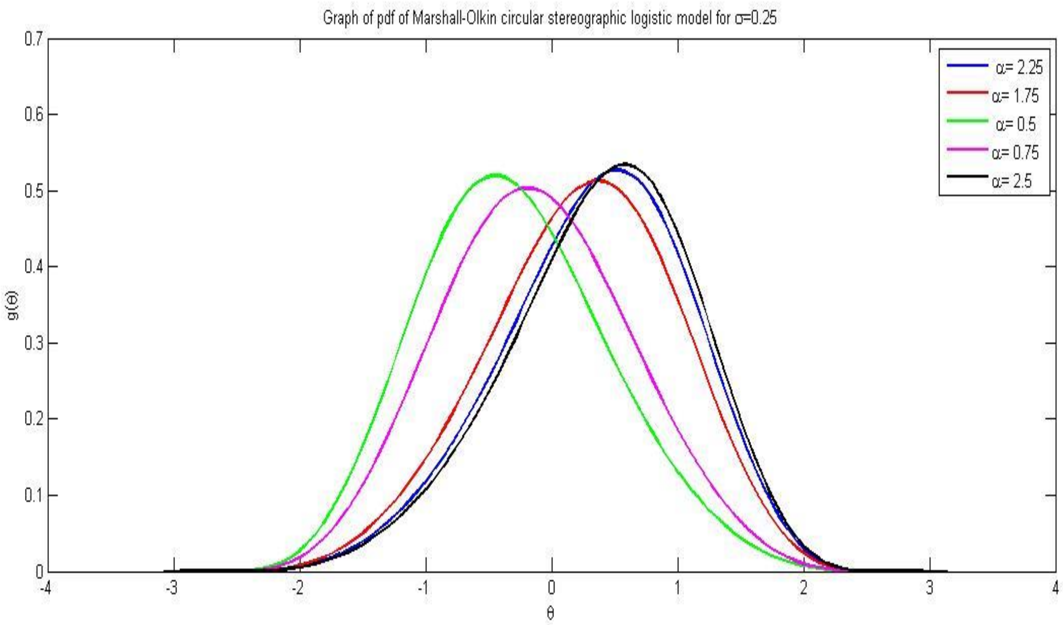
$$g(\theta) = \frac{\alpha}{2\sigma}\sec^2\left(\frac{\theta}{2}\right)\left[1+\alpha\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-2}\exp\left(-\left(\frac{\tan\left(\frac{\theta}{2}\right)-\mu}{\sigma}\right)\right), \qquad (3)$$

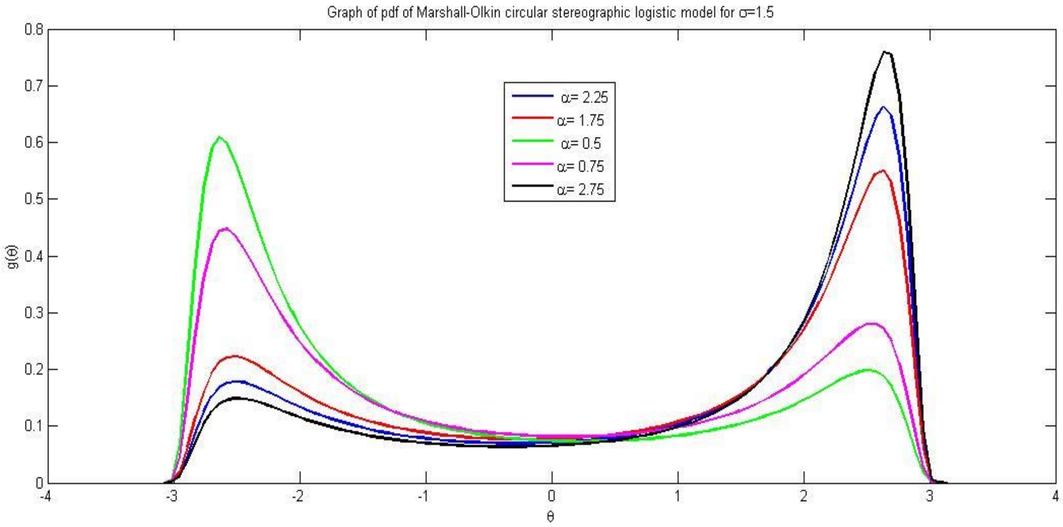
Where σ, $\alpha > 0$ and $-\pi \le \theta, \mu < \pi$

$$G(\theta) = \left[1+\alpha\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-1}, \text{where } \sigma, \alpha > 0,\ -\pi \le \theta < \pi. \qquad (4)$$

This distribution is asymmetric for $\alpha \neq 1$ and symmetric for $\alpha = 1$. Marshall-Olkin Stereographic logistic distribution is **unimodal** if $\sigma < 0.5$ and **bimodal** if $\sigma > 0.5$

Graphs of probability density function of Marshall-Olkin Stereographic Circular Logistic Distribution for various values of σ, α and $\mu = 0$ are presented here.





4. Extension to l-axial Marshall-Olkin Stereographic Circular Logistic Distribution

We extend the proposed model to the l -axial distribution, which is applicable to any arc of arbitrary length say π/l for $l=1,2,...,$ so it is desirable to extend the Marshall-Olkin stereographic circular logistic distribution. To construct the l -axial Marshall-Olkin stereographic circular logistic distribution, we consider the density function of Marshall-Olkin stereographic circular logistic distribution and use the transformation $\phi=\theta/l$, $l=1,2,...,$. The probability density function of ϕ is given by

$$g(\phi)=\frac{\alpha l}{2\sigma}\sec^2\left(\frac{l\phi}{2}\right)\left[1+\alpha e^{-\frac{\tan\left(\frac{l\phi}{2}\right)}{\sigma}}\right]^{-2} e^{-\frac{\tan\left(\frac{l\phi}{2}\right)}{\sigma}} \text{, where } -\frac{\pi}{l}<\phi<\frac{\pi}{l} \tag{5}$$

Case (1) when $l=1$, the probability density function (6.1) is the same as that of Marshall-Olkin Stereographic Circular Logistic Distribution.

$$g(\phi)=\frac{\alpha}{2\sigma}\sec^2\left(\frac{\phi}{2}\right)\left[1+\alpha e^{-\frac{\tan\left(\frac{\phi}{2}\right)}{\sigma}}\right]^{-2} e^{-\frac{\tan\left(\frac{\phi}{2}\right)}{\sigma}}, \text{ where } -\pi<\phi<\pi \tag{6}$$

Case (2) When $l=2$, the probability density function (6.1), the probability density function is

$$g(\phi)=\frac{\alpha}{\sigma}\sec^2(\phi)\left[1+\alpha e^{-\frac{\tan(\phi)}{\sigma}}\right]^{-2} e^{-\frac{\tan(\phi)}{\sigma}}, \text{ where } -\frac{\pi}{2}<\phi<\frac{\pi}{2} \tag{7}$$

It is called as Marshall-Olkin Stereographic Semicircular Logistic Distribution.

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